1. Geometrical optics

- is an approximation for $\lambda \to 0$
  no diffraction effects observed

Postulates

- light is emitted by sources
- light is detected by detectors
- light propagates in form of rays
- light matter interaction $\to n$, $n = \frac{c_0}{c}$
- reflected / refracted
- straight path for homog
- curved for inhomogeneous

E: lenses
shadow
pinhole camera

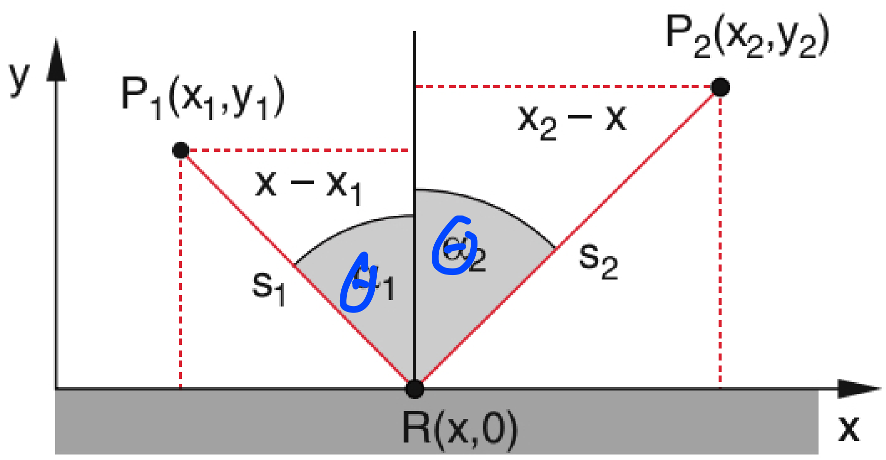
↳ difficult to describe:

- interference
- imaging
- diffraction

1.1. Reflection

$$\Theta_1 = \Theta_2$$

Fermat: light takes the fastest path



$$s = s_1 + s_2 = \sqrt{(x-x_1)^2 + y_1^2}$$
$$+ \sqrt{(x_2-x)^2 + y_2^2}$$

transit time $t = \frac{s}{c} \rightsquigarrow$ min

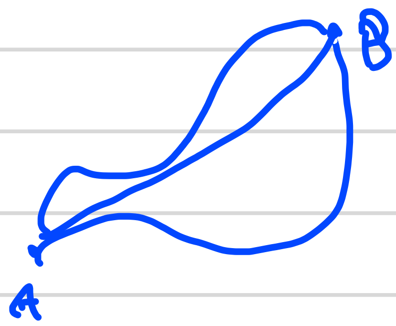
$$\frac{dt}{dx} = 0$$

$$\rightsquigarrow \frac{x - x_1}{\sqrt{(x-x_1)^2 + y_1^2}} = \frac{x_2 - x}{\sqrt{(x_2-x)^2 + y_2^2}}$$

$$\rightsquigarrow \sin\Theta_1 = \sin\Theta_2$$
$$\Theta_1 = \Theta_2$$

E: plane mirror

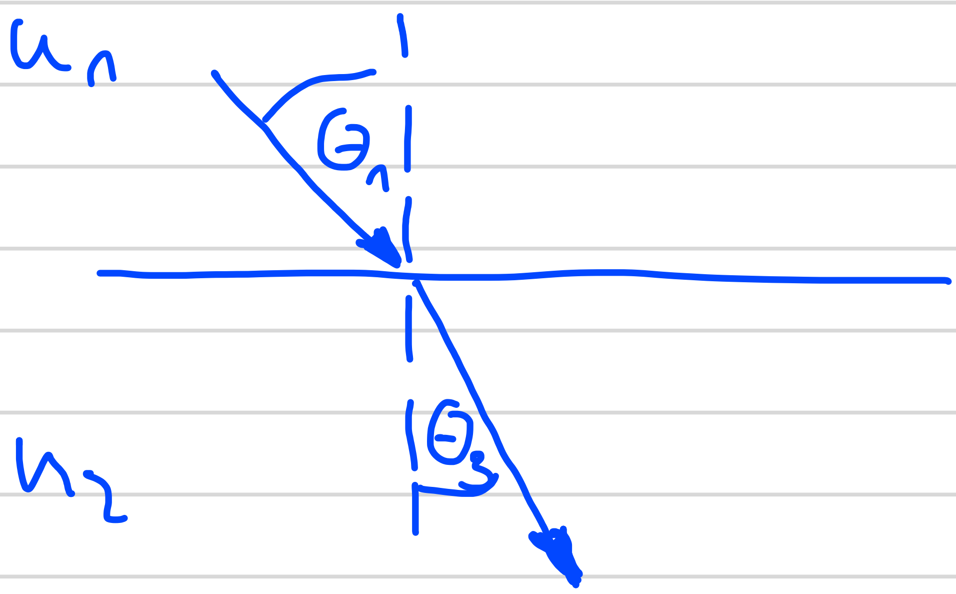
Fermat in general (also diff. version)



$$\delta \int_{P_1}^{P_2} n \cdot ds = 0$$

variational principle

1.2. Refraction



Snells law

$$n_1 \cdot \sin\Theta_1 = n_2 \sin\Theta_2$$

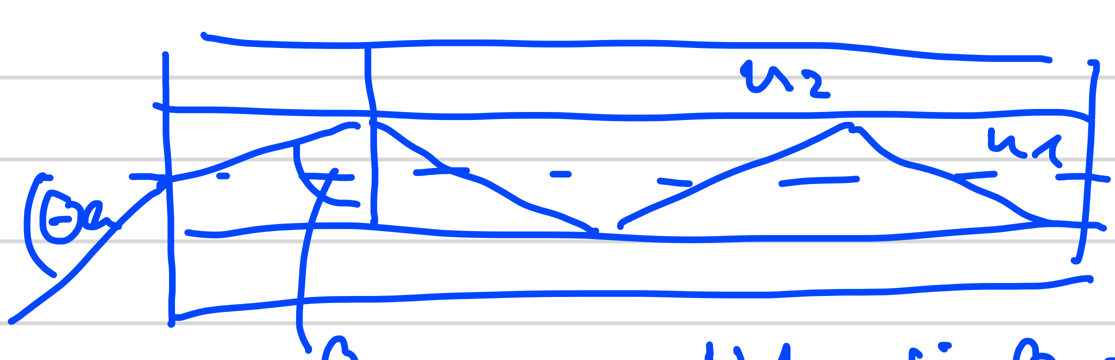
for $n_1 < n_2$
$\Theta_1 > \Theta_2$

for $n_1 > n_2$
$\Theta_1 < \Theta_2$

$\Rightarrow$ special case $\Theta_2 = \frac{\pi}{2}$

E: TIR
- Acrylic disk
- water beam
- beam of glass

total internal reflection: $\Theta_1 > \arcsin\left(\frac{n_2}{n_1}\right)$



$NA = \sin\Theta_a = \sqrt{n_1^2 - n_2^2}$

$NA \approx 0.2$, $n_1 = 1.475$, $n_2 = 1.46$

1.3. Mirrors, Prisms, Lenses

plane mirror: image formation

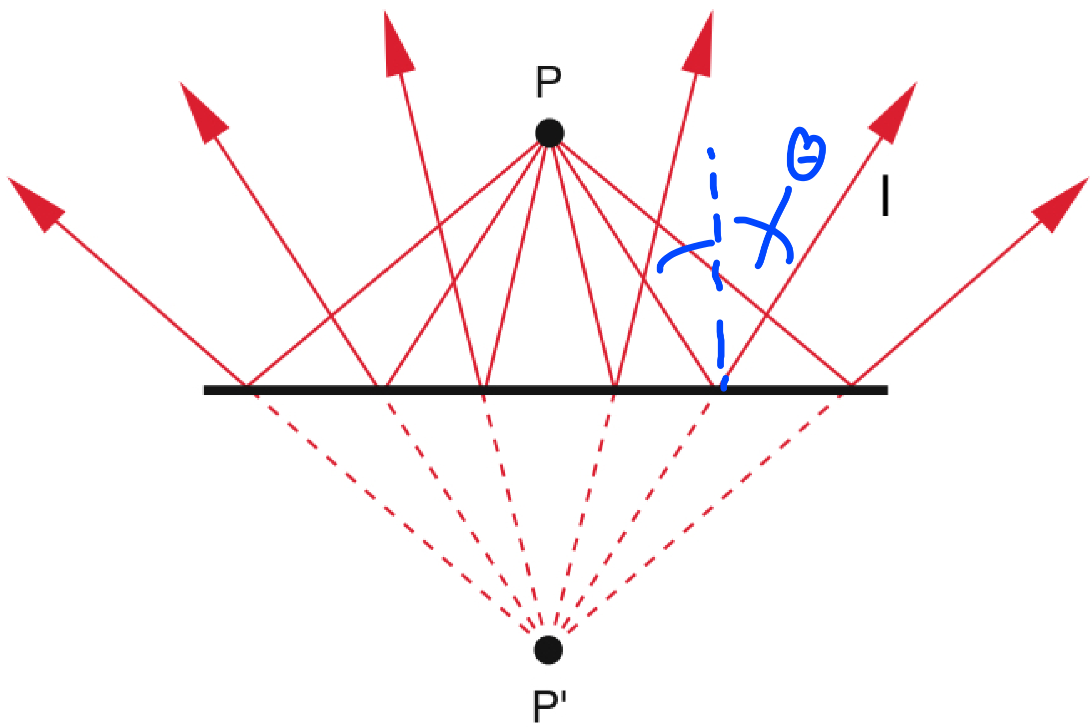


Fig. 9.4 Optical imaging by a plane mirror which produces from every arbitrary point above the mirror a virtual image below the mirror

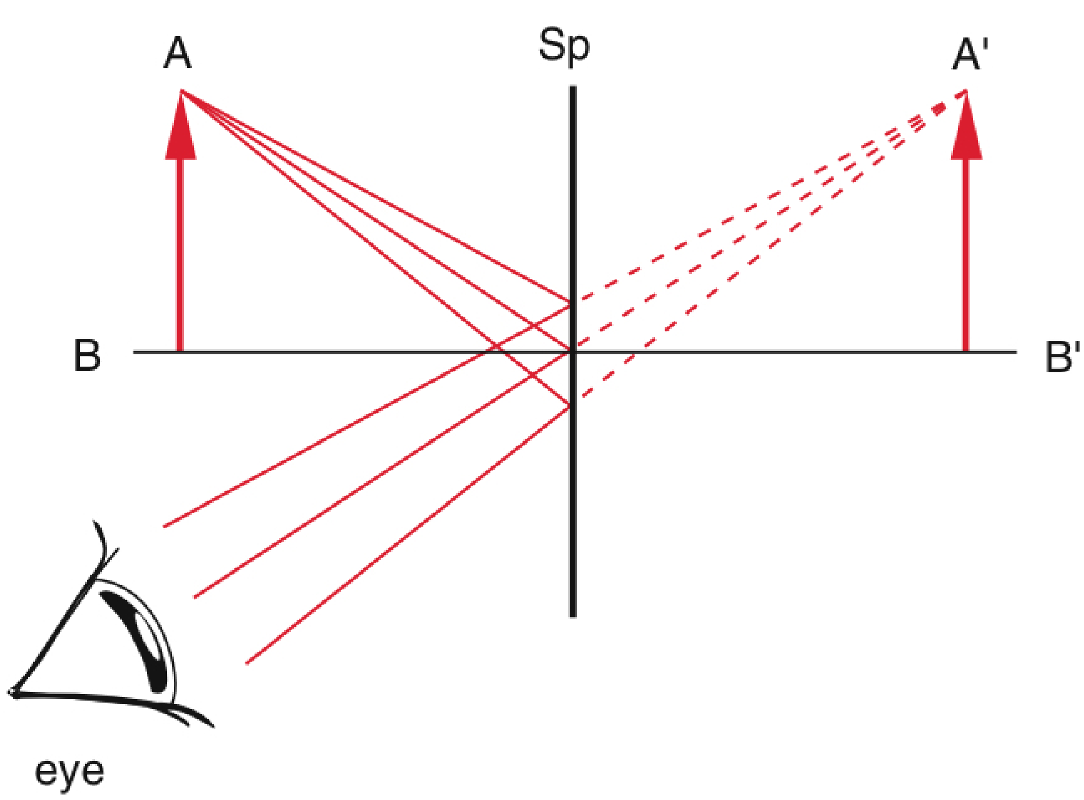
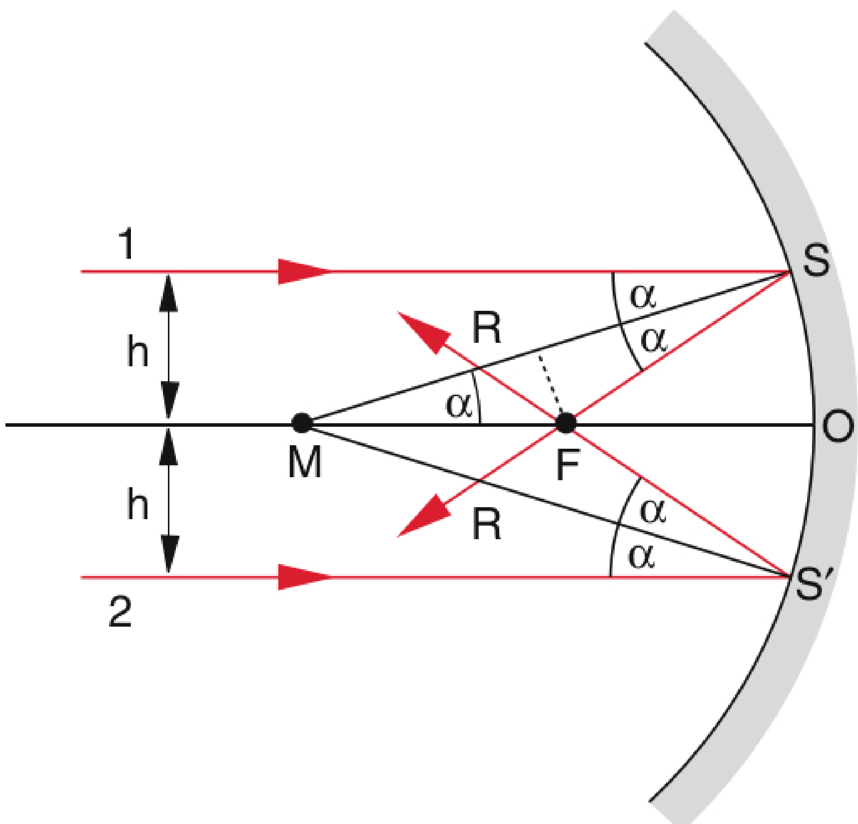


Fig. 9.5 A plane mirror images the object *AB* into the virtual image *A′ B′* of the same size (Magnification $M = 1$)

Concave mirrors



F … focal point
f … focal distance

$\alpha_i = \alpha_r = \alpha$

$FM = \frac{R}{2} / \cos(\alpha)$

$\rightsquigarrow$

$$\underline{OF = R\left(1 - \frac{1}{2\cos\alpha}\right)}$$

focal length

for small α $\Rightarrow$ $\cos\alpha \approx 1$

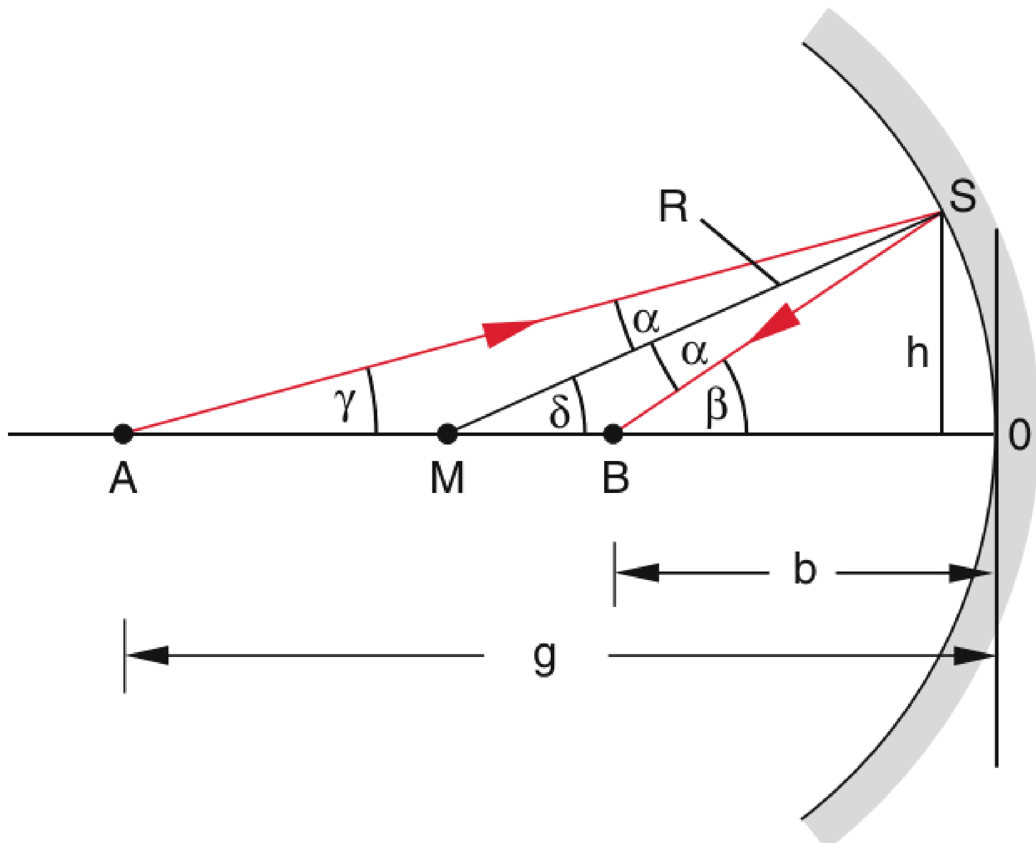
$$\rightsquigarrow f = OF \approx \frac{R}{2}$$

or $\cos\alpha = \sqrt{1 - \sin^2\alpha}$

$\sin\alpha = \frac{h}{R}$

$$\rightsquigarrow f = R\left[1 - \frac{R}{2\sqrt{R^2 - h^2}}\right]$$

focal length depends on h!



$\gamma + \alpha + \beta = 180°$

$\beta + \delta = 180$

$\gamma + \alpha + \cancel{180} - \delta = \cancel{180}$

$\delta = \alpha + \gamma$ (exterior angle

$\hookrightarrow \gamma + \beta = 2\delta$

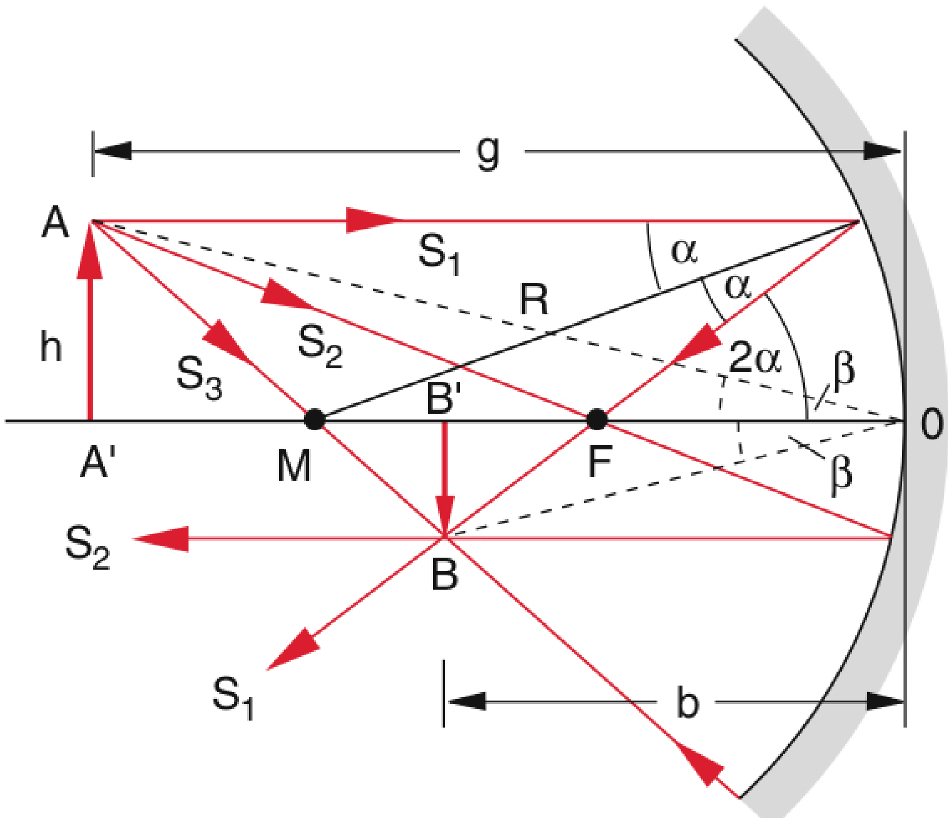
for small angles, small h

$\gamma \approx \tan\gamma = \frac{h}{g}$ ($\sin\gamma \approx \gamma$, $\cos\gamma \approx 1$)

$\beta \approx \tan\beta = \frac{h}{b}$

$\delta = \sin\delta = \frac{h}{R}$

$\rightarrow \boxed{\frac{1}{g} + \frac{1}{b} \approx \frac{2}{R} = \frac{1}{f}}$ imaging equation



do calculate that a bit

$$\frac{2}{f} + \frac{1}{b} = \frac{1}{g}$$

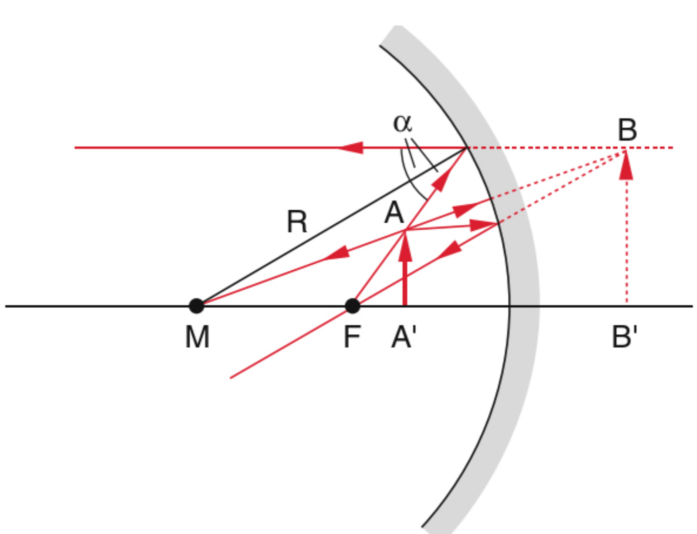
$$\frac{1}{f} = -\frac{1}{b} \qquad f = -b$$

1) $g > 2f$:
- image real
- image

2) $g = 2f$
- same size
- reversed

3) $f < g < 2f$
- larger image
- reversed

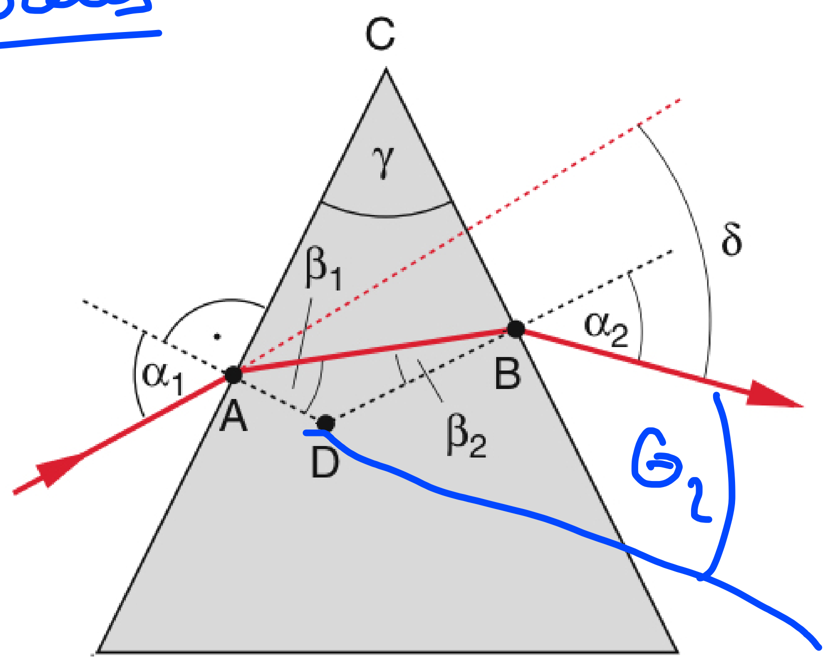
4) $g < f$
- larger image virtual



virtual image

Prisms

isosceles



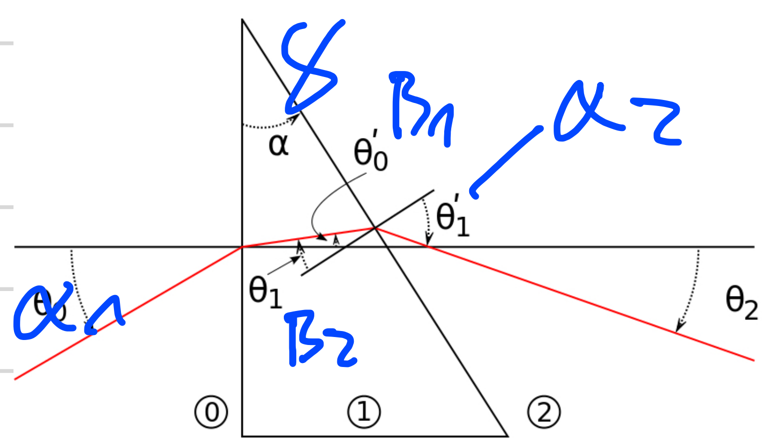
$$\delta = \alpha_1 - \beta_1 + \alpha_2 - \beta_2$$

$$\gamma = \beta_1 + \beta_2$$

$$\gamma + (90^\circ - \beta_1) + (90^\circ - \beta_2) = 180^\circ$$

$$\rightsquigarrow \delta = \alpha_1 + \alpha_2 - \gamma$$

$$\beta_1 = \sin^{-1}\left(\frac{n_0}{n_1} \sin(\alpha_1)\right)$$

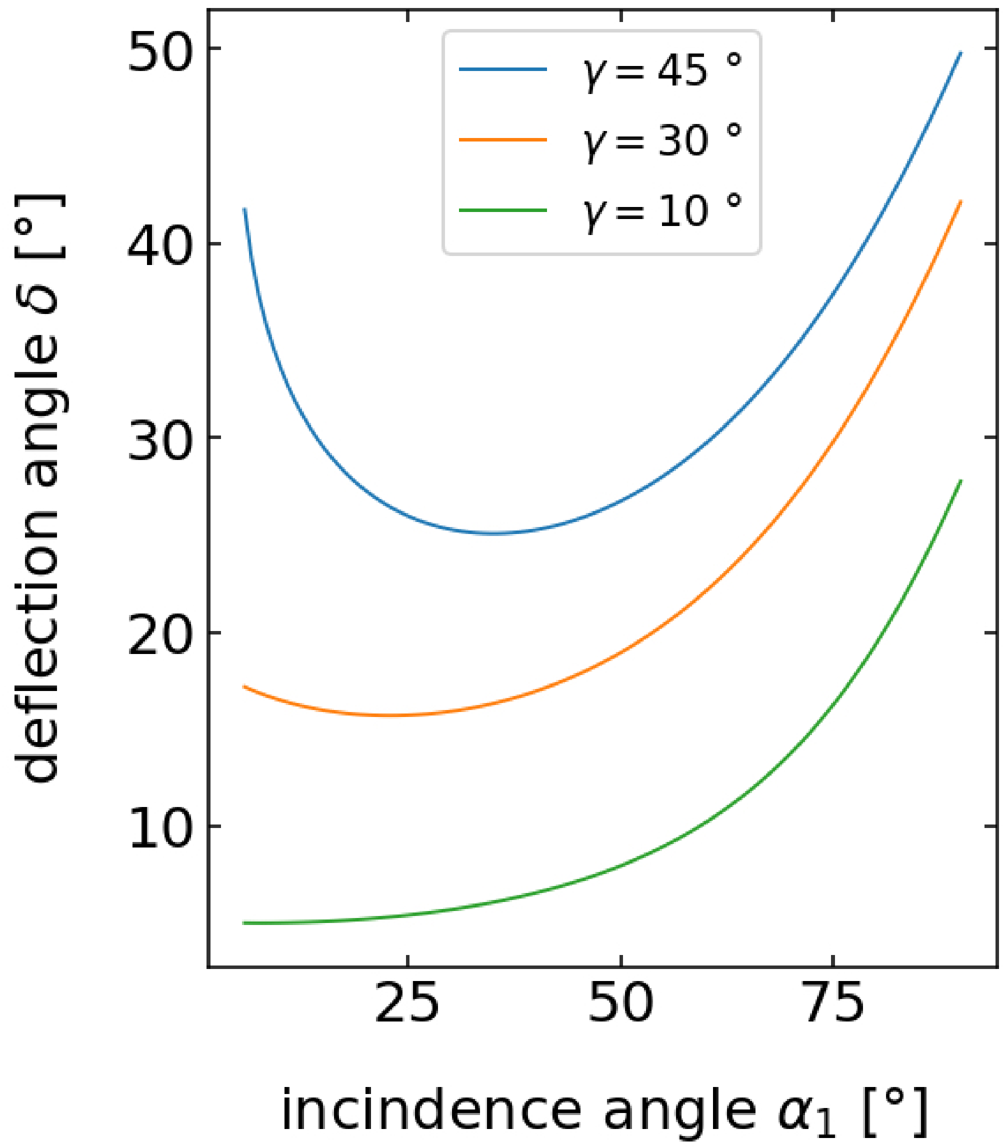


$$\beta_2 = \gamma - \beta_1$$

$$\alpha_2 = \sin^{-1}\left(\frac{n_1}{n_0} \sin(\beta_2)\right)$$

$$\theta_2 = \alpha_2 - \gamma$$

$$\rightsquigarrow \delta = \alpha_1 + \sin^{-1}\left(\frac{n_1}{n_0} \sin\left[\gamma - \sin^{-1}\left(\frac{n_0}{n_1} \sin(\alpha_1)\right)\right]\right)$$



$\rightsquigarrow$ $\delta = \alpha_1 + \alpha_2 - \gamma$

minimum deflection:

$$\frac{d\delta}{d\alpha_1} = 1 + \frac{d\alpha_2}{d\alpha_1} = 0$$

$\rightsquigarrow$ $d\alpha_2 = - d\alpha_1$ | $\rightarrow$ Snell law

$\sin\alpha_1 = n \sin\beta$

$\cos\alpha_1 d\alpha_1 = n \cos\beta_1 d\beta_1$

$$\rightarrow \frac{1-\sin^2\alpha_1}{1-\sin^2\alpha_2} = \frac{n^2 - \sin^2\alpha_1}{n^2 - \sin^2\alpha_2}$$

$\cos\alpha_2 d\alpha_2$

for $n \neq 1$ $\rightsquigarrow$ $\alpha_1 = \alpha_2$

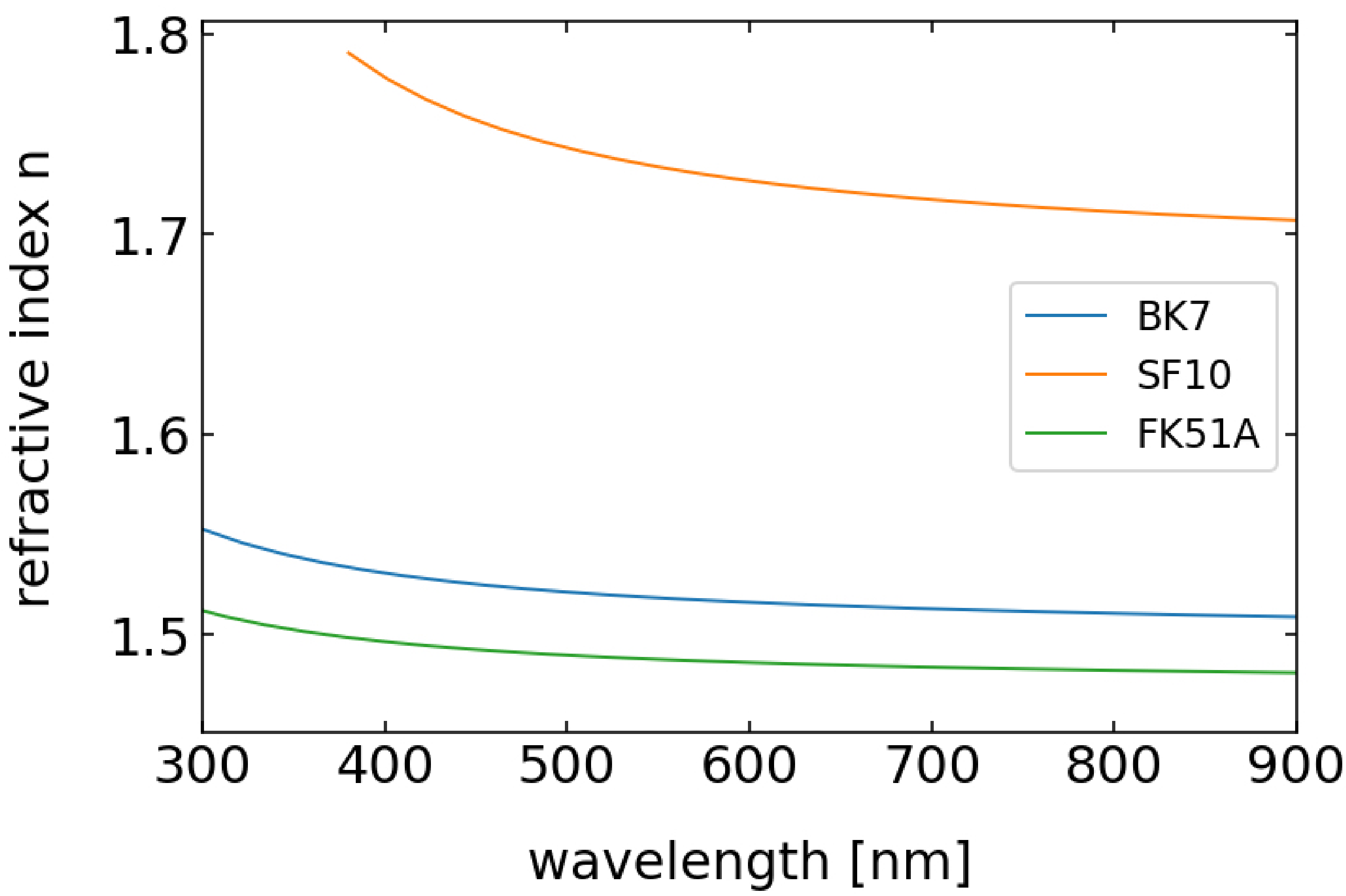
$n \cos\beta_2 d\beta_2$

For the symmetric ray path with $AC = BC$ and $\alpha_1 = \alpha_2$ the deflection δ is minimum. For the incident angle α the total deflection δ of rays passing through an isosceles prism with prism angle γ is

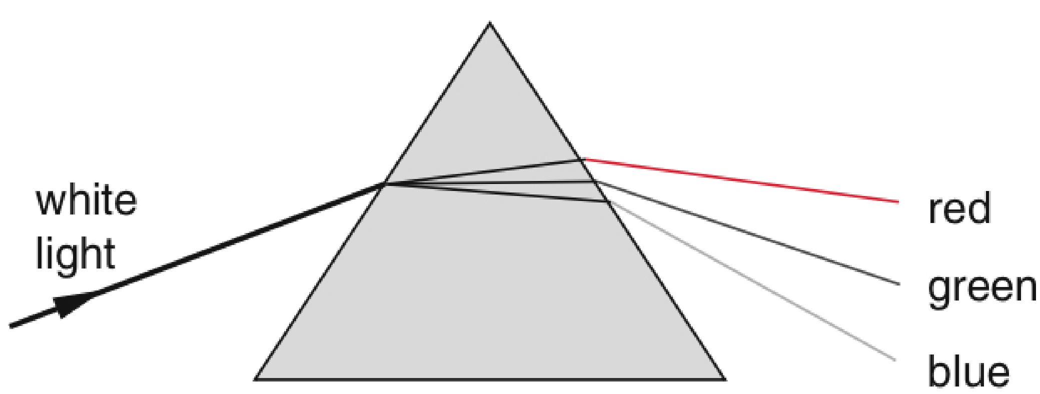
$$\delta_{\min} = 2\alpha - \gamma. \tag{9.17}$$

with $\sin\alpha = n \sin\beta$

$$\sin\frac{\delta_{\min}+\gamma}{2} = \sin\alpha = n\cdot\sin\beta$$
$$= n\cdot\sin(\gamma/2)$$



n is a fnc of λ (dispersion)



normal
$\frac{dn}{d\lambda} < 0$

anomalous
$\frac{dn}{d\lambda} > 0$

Fig. 9.20 Within the spectral range of normal dispersion ($dn/d\lambda < 0$) blue light is deflected more than red light

$$\frac{d\delta}{dn} = \frac{2\sin(\gamma/2)}{\sqrt{1 - n^2 \sin^2(\gamma/2)}}$$

$$\frac{d\delta}{d\lambda} = \frac{d\delta}{dn}\frac{dn}{d\lambda} \quad \text{with } n(\lambda)$$

$$\frac{d\delta}{d\lambda} = \frac{2\sin(\gamma/2)}{\sqrt{1-n^2\sin^2(\gamma/2)}}\frac{dn}{d\lambda}$$

for most transparent media

$$\frac{dn}{d\lambda} < 0$$

Example

For an isosceles prism with ($\gamma = 60°$) is

$$\frac{d\delta}{d\lambda} = \frac{dn/d\lambda}{\sqrt{1-n^2/4}}.$$

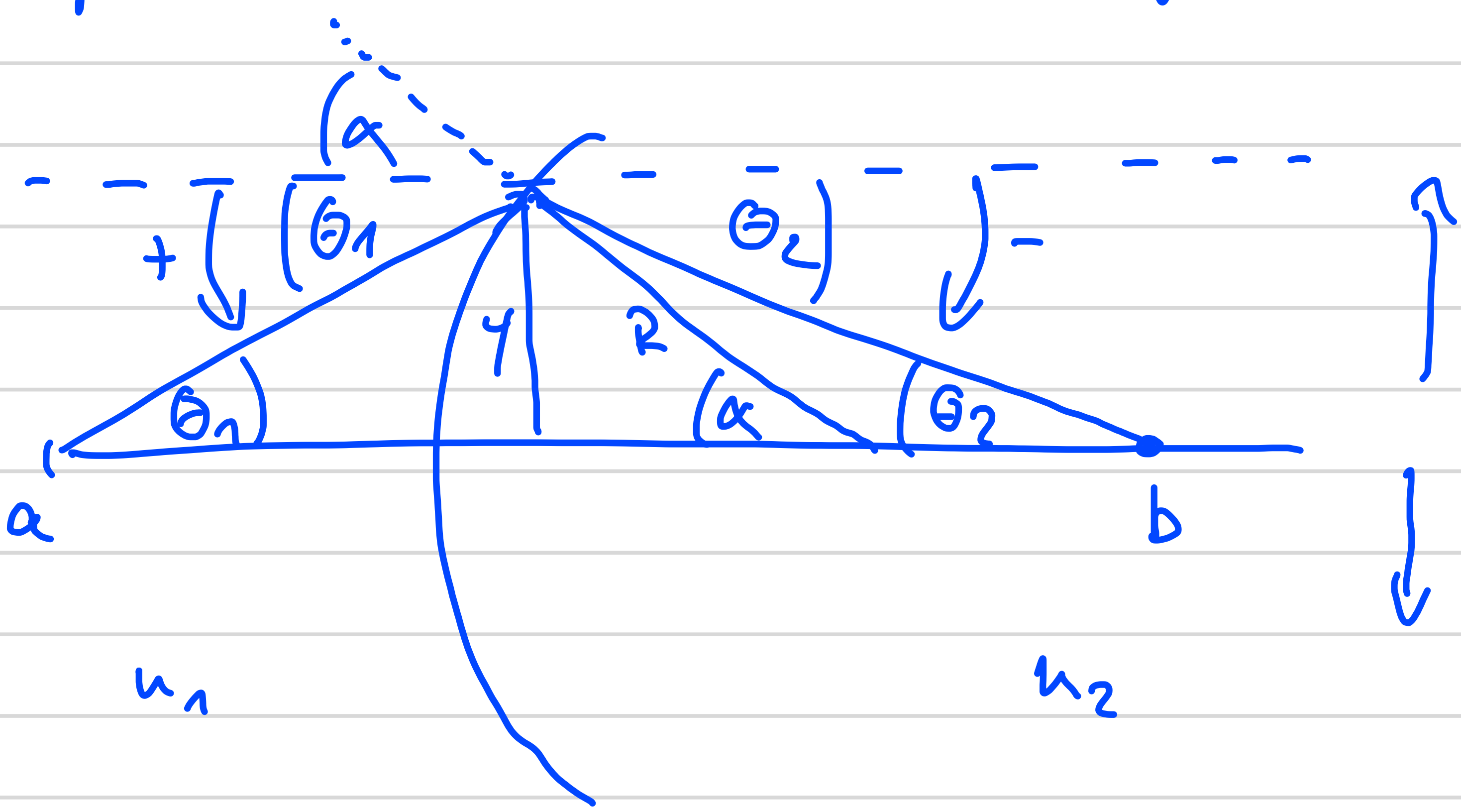
With $dn/d\lambda = 4 \times 10^5$ m^{-1} at the wavelength $\lambda = 400$ nm and $n = 1.8$ (for Flint glass) we obtain $d\delta/d\lambda = 1 \times 10^3$ rad/nm. Two wavelengths λ_1 and λ_2 which differ by $\Delta\lambda = 10$ nm experience deflection angles that differ by 10^{-2} rad $\approx 0.6°$.

↪ spectroscopy

↪ DIY prism

Lenses

Refraction at curved surfaces



$$n_1 \sin(\alpha + \theta_1) = n_2 \sin(\alpha + \theta_2)$$

$$\sin(\alpha) = \frac{y}{R}, \quad \tan(\theta_1) = \frac{y}{a}, \quad \tan(\theta_2) \approx \frac{y}{b}$$

linearization:

$$\sin(\theta) \approx \theta$$

$$\rightsquigarrow \quad n_1(\alpha + \theta_1) = n_2(\alpha - \theta_2)$$

$$n_1 \alpha + n_1 \theta_1 = n_2 \alpha - n_2 \theta_2$$

$$(n_1 - n_2)\alpha + n_1 \theta_1 = -n_2 \theta_2$$

$$n_2 \theta_2 = -(n_1 - n_2)\alpha - n_1 \theta_1$$

$$\theta_2 = \frac{n_2 - n_1}{n_2} \alpha - \frac{n_1}{n_2} \theta_1$$

$$\theta_2 = \frac{n_2 - n_1}{n_2}\,\alpha - \frac{n_1}{n_2}\,\theta_1$$

$$\theta_2 = \frac{n_2 - n_1}{n_2}\,\frac{y}{R} - \frac{n_1}{n_2}\,\theta_1$$

for $\theta_1 = 0$ $\Rightarrow$ $\theta_2 = \frac{n_2 - n_1}{n_2}\,\frac{y}{R}$

$$\theta_2 \approx \frac{\Delta y + y}{b}$$

$$\frac{\Delta y + y}{b} = \frac{n_2 - n_1}{n_2 R}\,y$$

$$\Delta y + y = \frac{n_2 - n_1}{n_2 R}\,y\,b$$

$\underline{y \approx 0:}$ auch $\quad n_2 \frac{\Delta y}{b} = n_1 \frac{y}{a} \Rightarrow \Delta y = \frac{n_1}{n_2}\,\frac{y}{a}\,b$

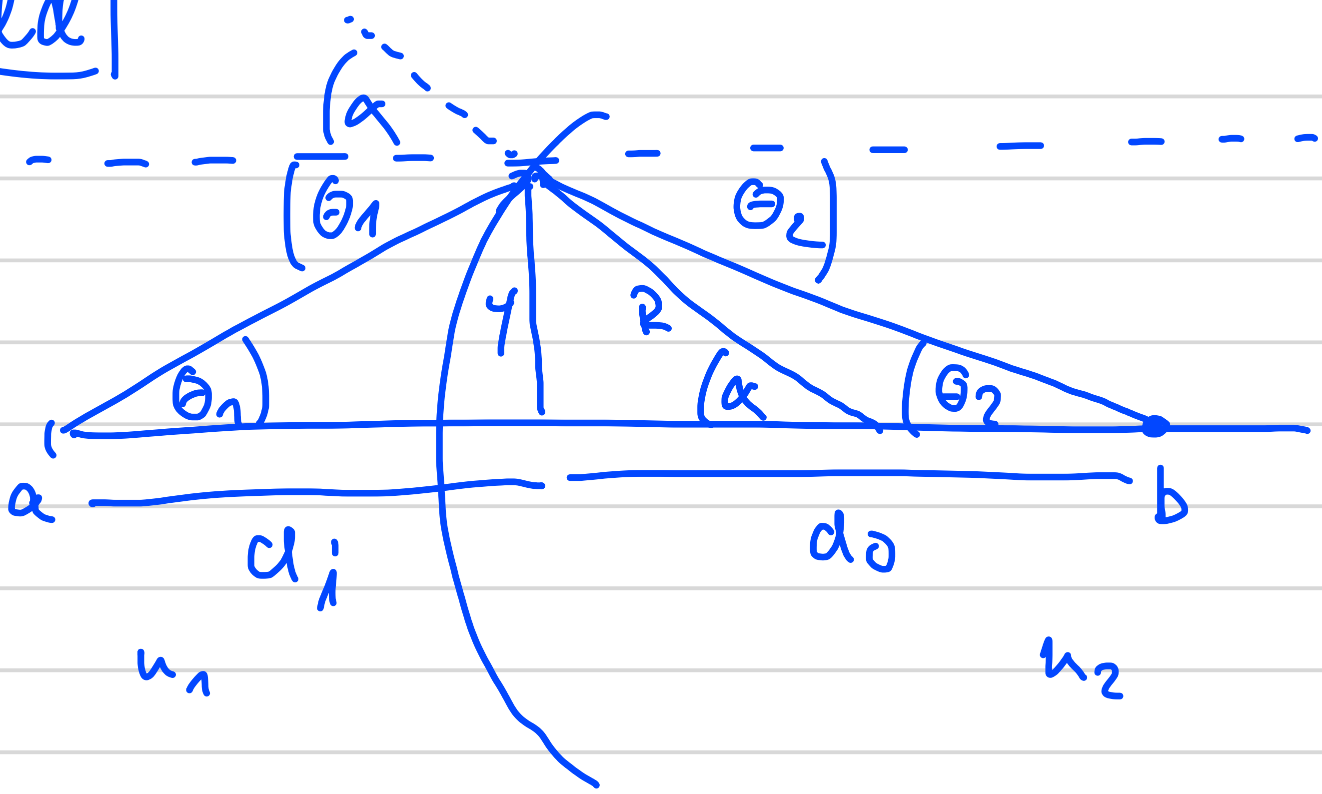
$$y\left(\frac{n_1 b}{n_2 a} + 1\right) = \frac{n_2 - n_1}{n_2 R}\,y\,b$$

$$y\left(-\frac{n_1}{n_2}\,\frac{b}{a} + 1\right) = \frac{n_2 - n_1}{n_2 R}\,\cancel{y}\,b$$

$$\frac{n_1}{n_2}\,\frac{b}{a} + 1 = \frac{n_2 - n_1}{n_2 R}\,b$$

$$\frac{n_1}{a} + \frac{n_2}{b} = \frac{n_2 - n_1}{R}$$

add



$$n_1 \sin(\theta_1 + \alpha) = n_2 (\alpha - \theta_2)$$

$$n_1 (\theta_1 + \alpha) \approx n_2 (\alpha - \theta_2)$$

$$\tan \theta_1 = \frac{y}{d_i} \qquad \tan \theta_2 = \frac{y}{d_o}, \qquad \tan \alpha = \frac{y}{R}$$

$$n_1 \left(\frac{y}{d_i} + \frac{y}{R} \right) = n_2 \left(\frac{y}{R} - \frac{y}{d_o} \right)$$

$$\frac{n_1}{d_i} + \frac{n_2}{d_o} = \frac{n_2 - n_1}{R}$$

$$\frac{b}{a} + 1 = \frac{n_2 - n_1}{n_2 R} b$$

$$\frac{n_1}{a} + \frac{n_2}{b} = \frac{n_2 - n_1}{R} = \frac{n_2}{f}$$ imaging equation.

$$f = \frac{n_2}{n_2 - n_1} R$$ focal length

$$\theta_t = \frac{y}{f_1} - \frac{n_1}{n_2}\theta_1 \qquad f_1 = \frac{n_2}{n_2 - n_1} R_1$$

$$\theta_2 = \frac{y}{f_2} - \frac{n_2}{n_1}\theta_t \ , \quad f_2 = \frac{n_1}{n_1 - n_2} R_2$$

$$\theta_2 = \frac{y}{f_2} - \frac{n_2}{n_1}\left(\frac{y}{f_1} - \frac{n_1}{n_2}\theta_1\right)$$

$$= \frac{y}{f_2} - \frac{n_2}{n_1}\left(\frac{(n_2 - n_1) y}{n_2 R_1} - \frac{n_1}{n_2}\theta_1\right)$$

$$= \frac{y}{f_2} - \frac{(n_2 - n_1) y}{n_1 R_1} + \theta_2$$

$$= \frac{n_1 - n_2}{n_1 R_2} y + \frac{n_1 - n_2}{n_1}\frac{y}{R_1} + \theta_1$$

$$\Theta_2 = \frac{n_1 - n_2}{n_1 R_2} y + \frac{n_1 - n_2}{n_1} \frac{y}{R_1} + \Theta_1$$

$$= \frac{n_1 - n_2}{n_1} \left(\frac{1}{R_2} + \frac{1}{R_1} \right) y + \Theta_1$$

$$\frac{1}{f} = \frac{n_1 - n_2}{n_1} \left(\frac{1}{R_2} + \frac{1}{R_1} \right)$$

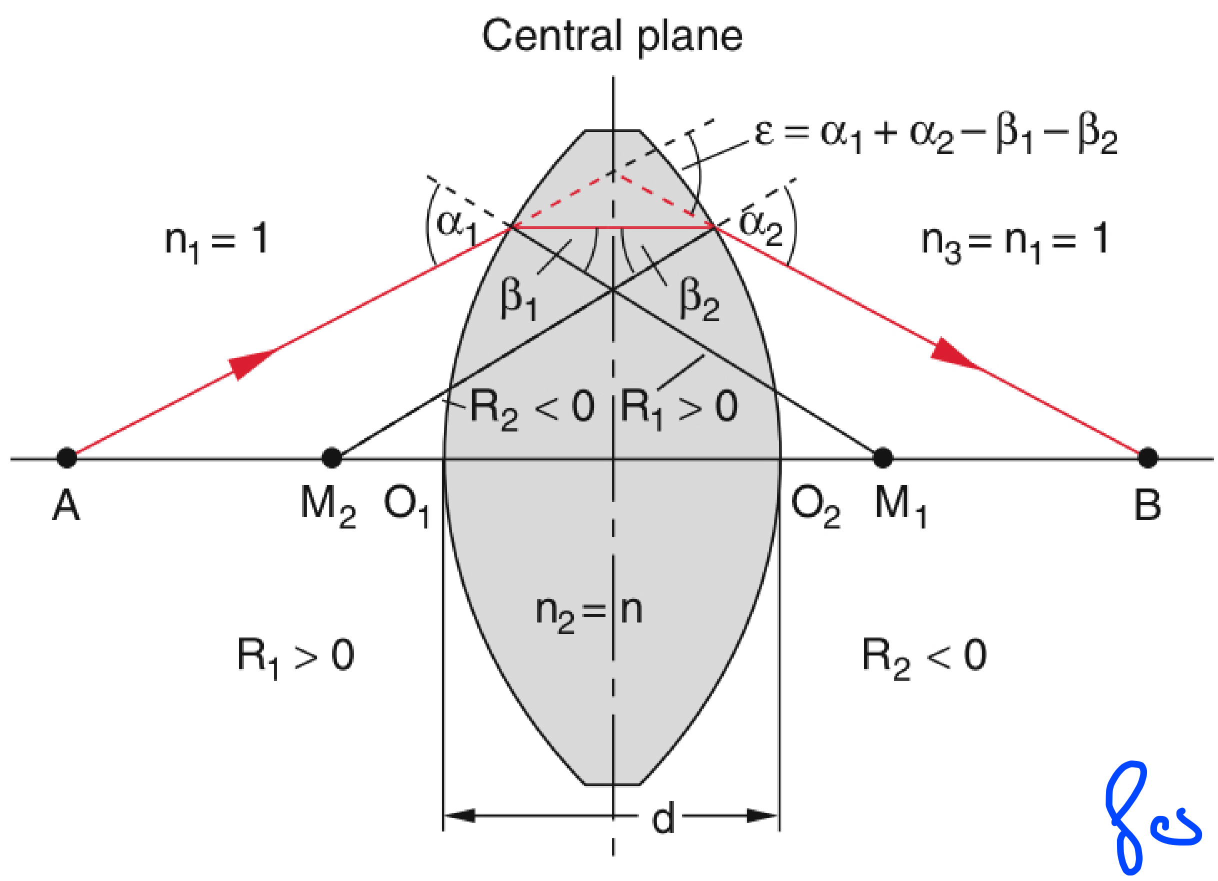
$$= \frac{n_1 - n_2}{n_1} \left(\frac{R_1 + R_2}{R_1 \cdot R_2} \right)$$

$$f = \frac{n_1}{n_1 - n_2} \left(\frac{R_1 R_2}{R_1 + R_2} \right)$$

$\longrightarrow$ leads to

thin lens

for a thin lens we apply that twice



for $n_1 = 1$

$$\frac{1}{a} + \frac{1}{b} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) = \frac{1}{f}$$

$$\Rightarrow f = \frac{1}{n-1}\left(\frac{R_1 \cdot R_2}{R_2 - R_1}\right)$$ lens maker equation

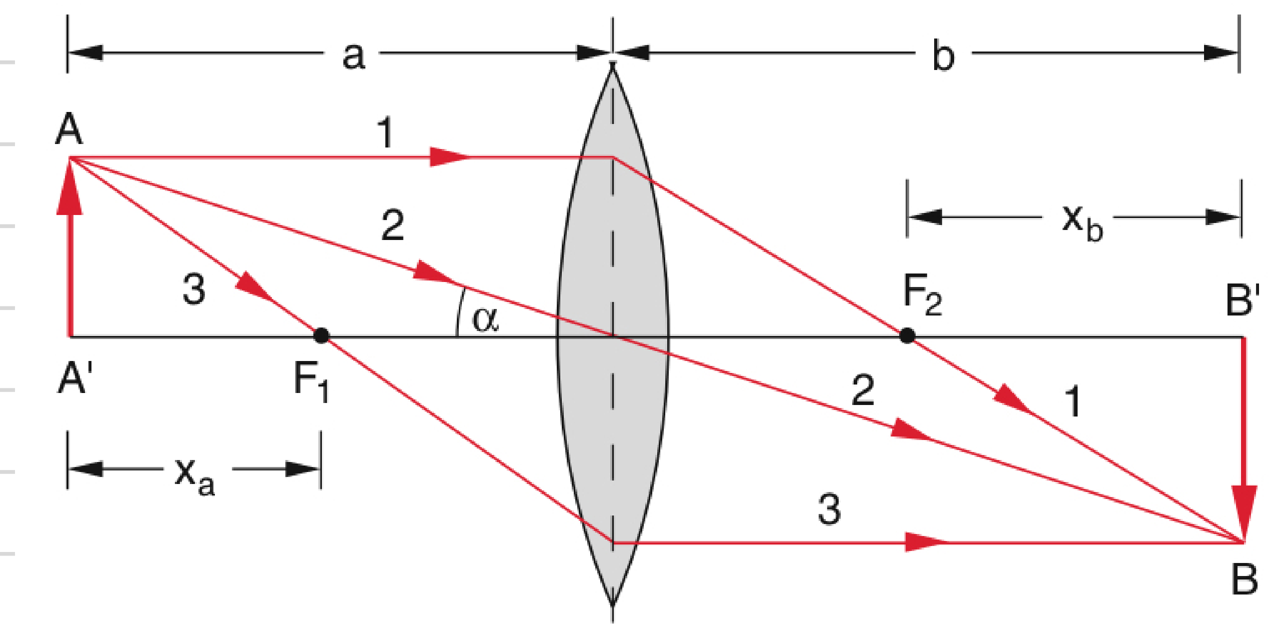
for a biconvex lens $R_1 = -R_2 = R$

$$f = \frac{R/2}{n-1}$$

$\frac{1}{f}$ dpt Diopter $1\,\text{dpt} = \frac{1}{m}$

geometrical construction

use 3 rays: parallel, central, focal ray



$$\frac{1}{x_a+f} + \frac{1}{x_b+f} = \frac{1}{f}$$

$$\cancel{1+}\ \frac{x_a+f}{x_b+f} = \frac{x_a+f}{f} = \frac{x_a}{f} \cancel{+1}$$

$$x_a + f = \frac{x_a x_b}{f} + \frac{x_a f}{f}$$

$x_a \cdot x_b = f^2$ Newton's imaging eq.

$$M = -\frac{b}{a} = \frac{f}{f-a}$$ magnification

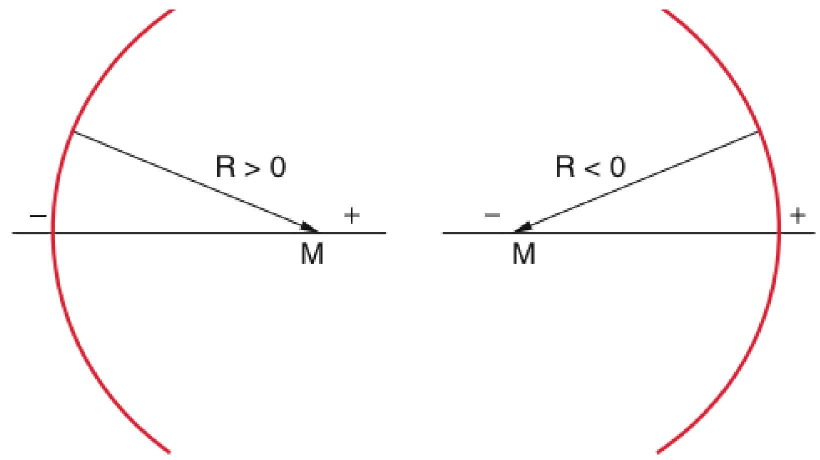
for $M<0$ object reversed

for $M>0$ object upright

- $a<f$ upright, mag., virtual
- $a>f$ reversed, real
- $f<a<2f$ mag
- $2f<a$ shrinked
- $a=f$ $M=\infty$, $b \to \infty$

do some of the calculation

definition of the radii of curvature



different lens types

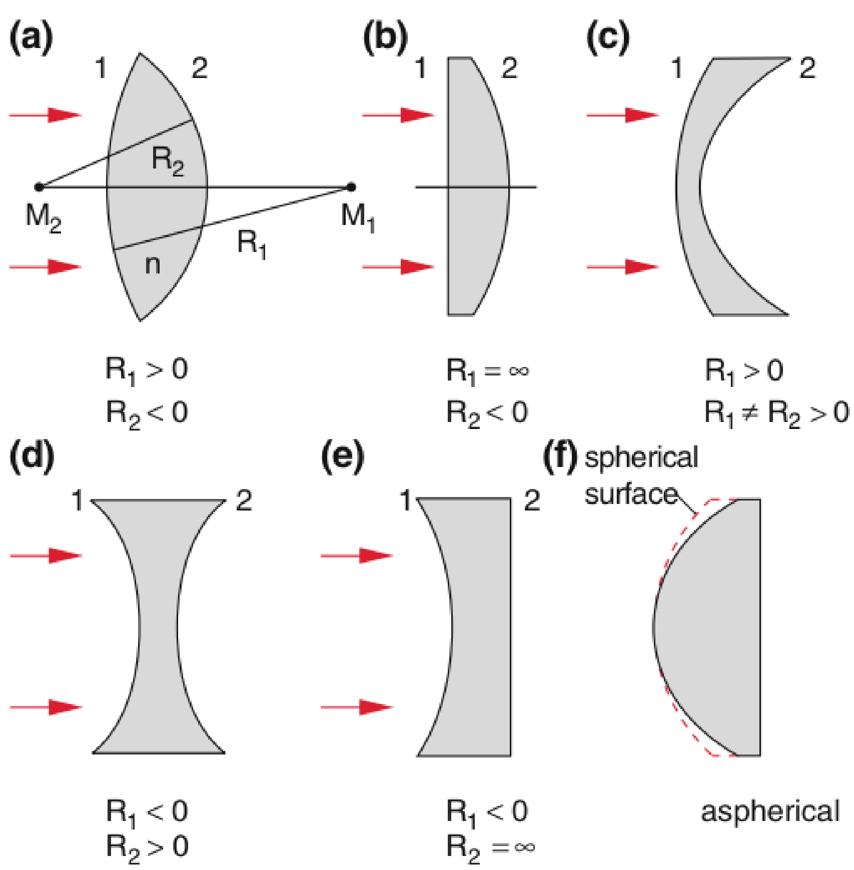
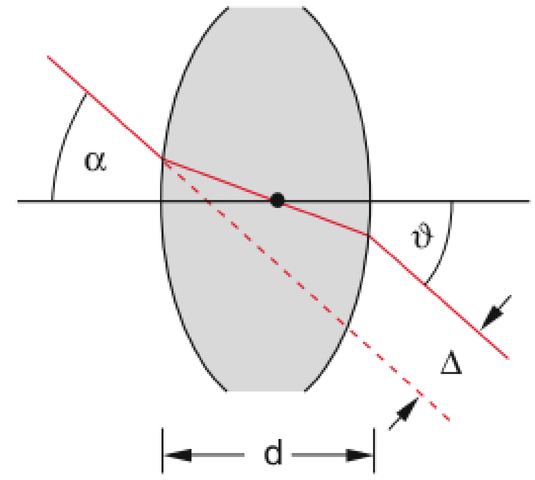


Fig. 9.26 Examples of different forms of lenses: **a**) convex-convex = biconvex **b**) plane-convex **c**) convex-concave **d**) biconcave **e**) concave-plane **f**) aspherical lens

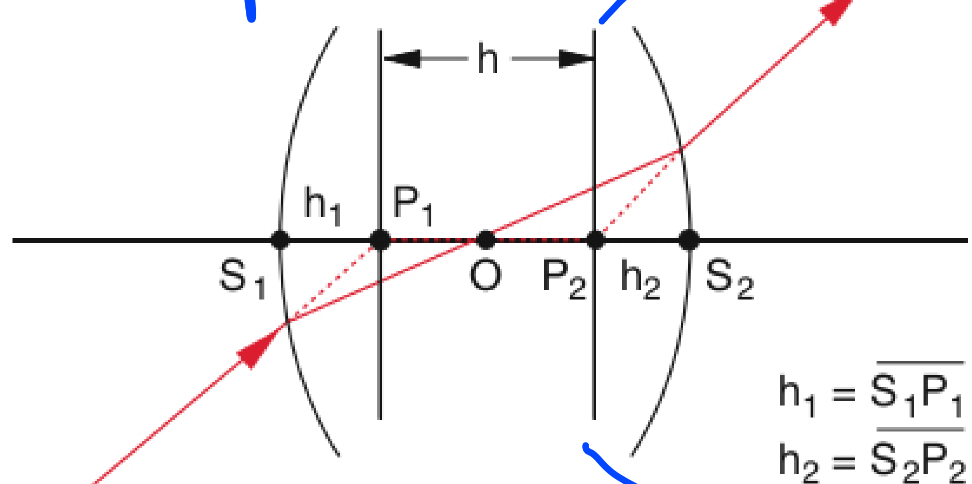
Thick lens



Substitute ray path

prisms

principle planes



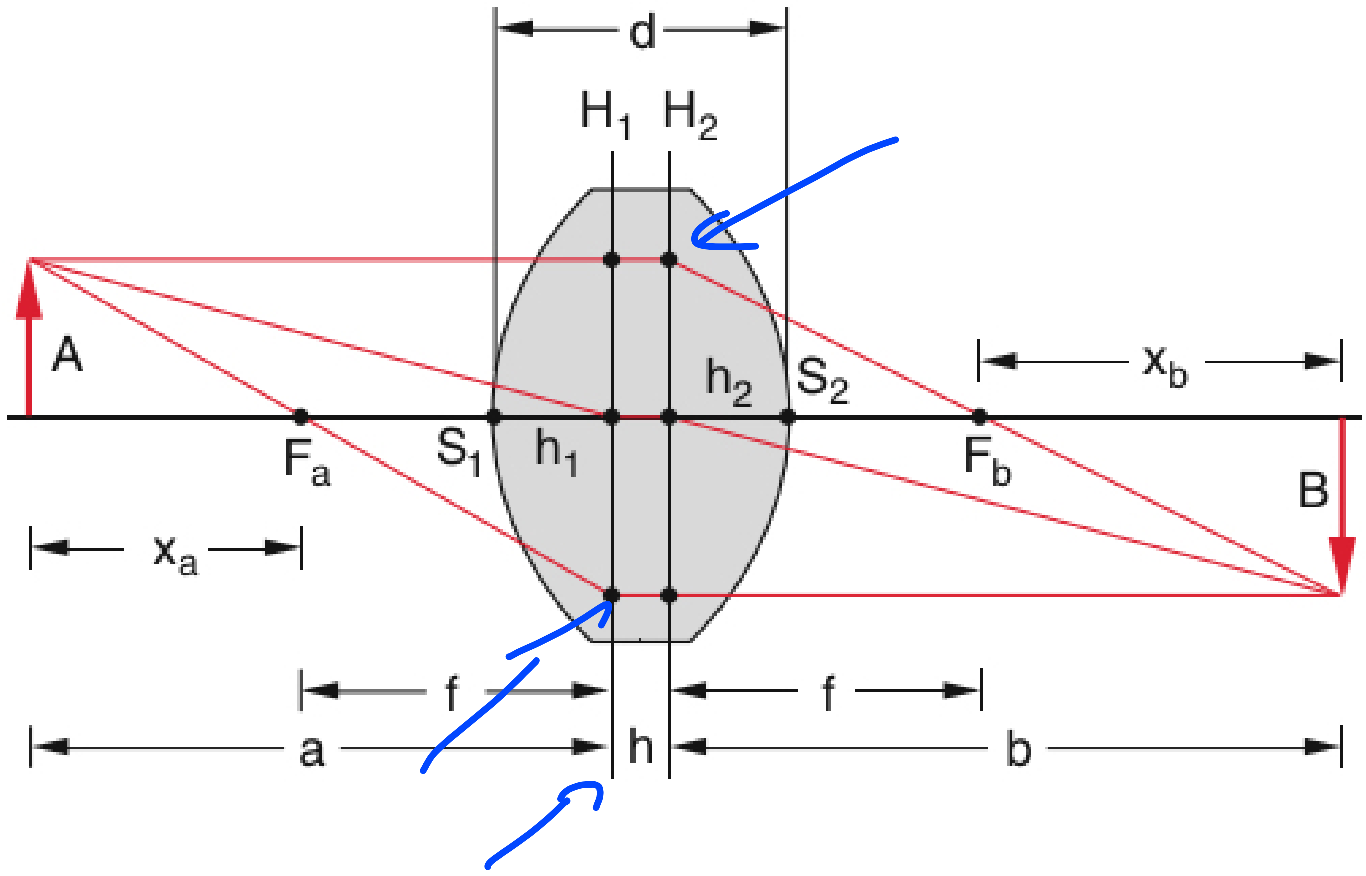
secondary

$$\frac{1}{f} = (n-1)\left[\frac{1}{R_1} - \frac{1}{R_2} + \frac{(n-1)d}{nR_1R_2}\right]$$

$$d = h_1 + h + h_2$$

$$h_1 = -\frac{(n-1)f\cdot d}{nR_2}$$

$$h_2 = -\frac{(n-1)f\cdot d}{nR_1}$$



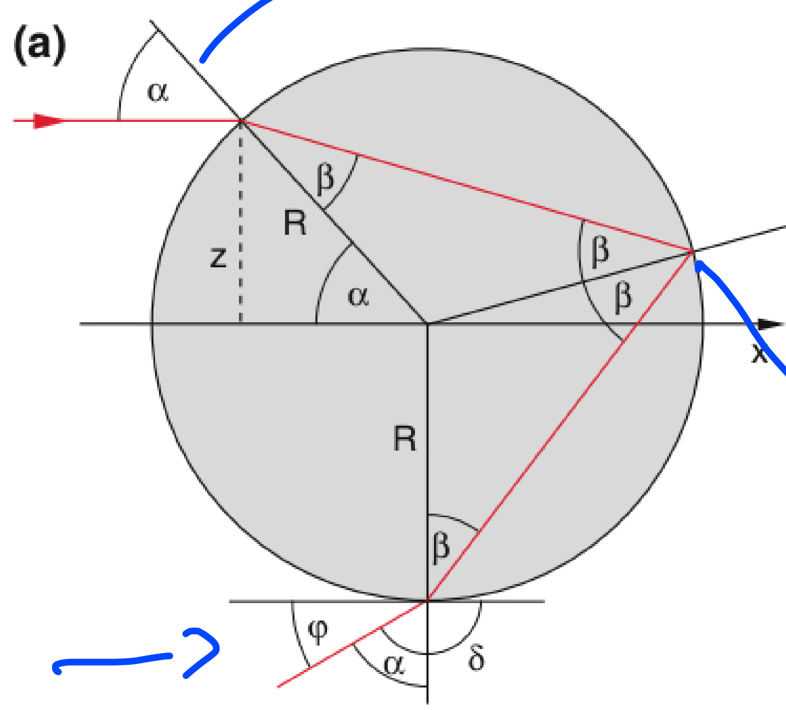
→ simple equations still valid

$$\frac{1}{a} + \frac{1}{b} = \frac{1}{f}$$

Rainbow

single drop

incident angle



outgoing angle

transmitted

deflection angle

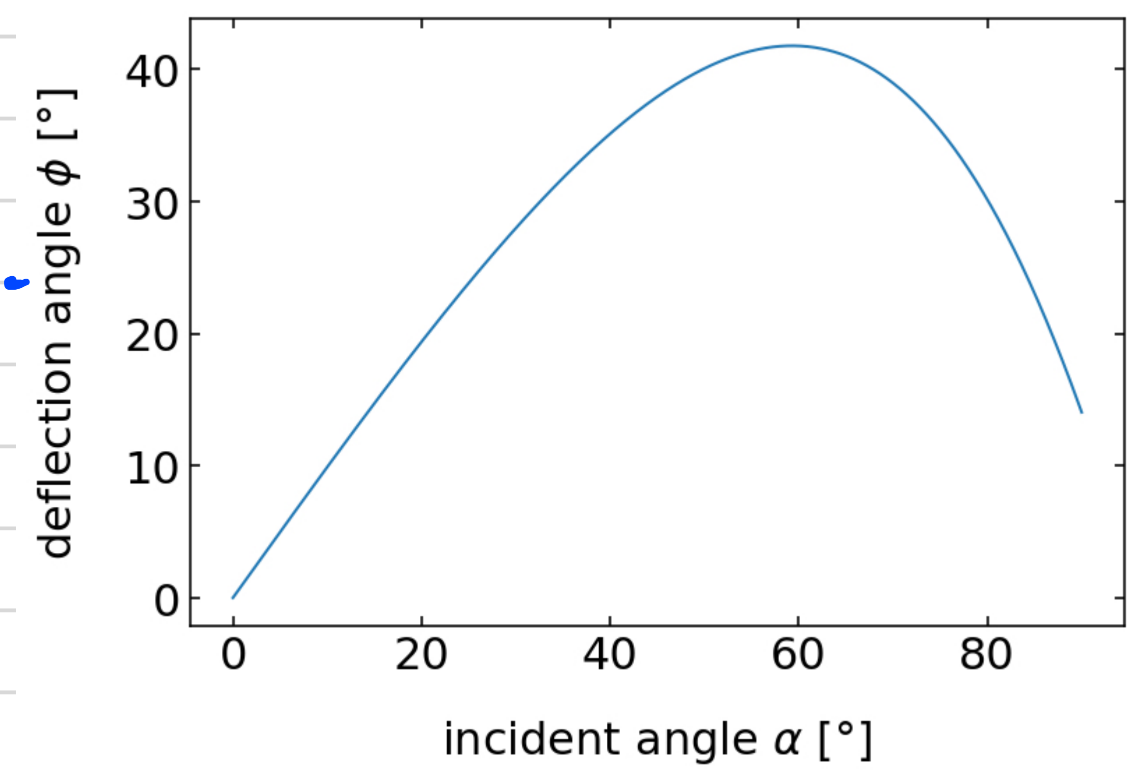
$$\delta = 180^\circ - \varphi = 180^\circ - 4\beta + 2\alpha$$

$$\varphi = 4\beta - 2\alpha$$

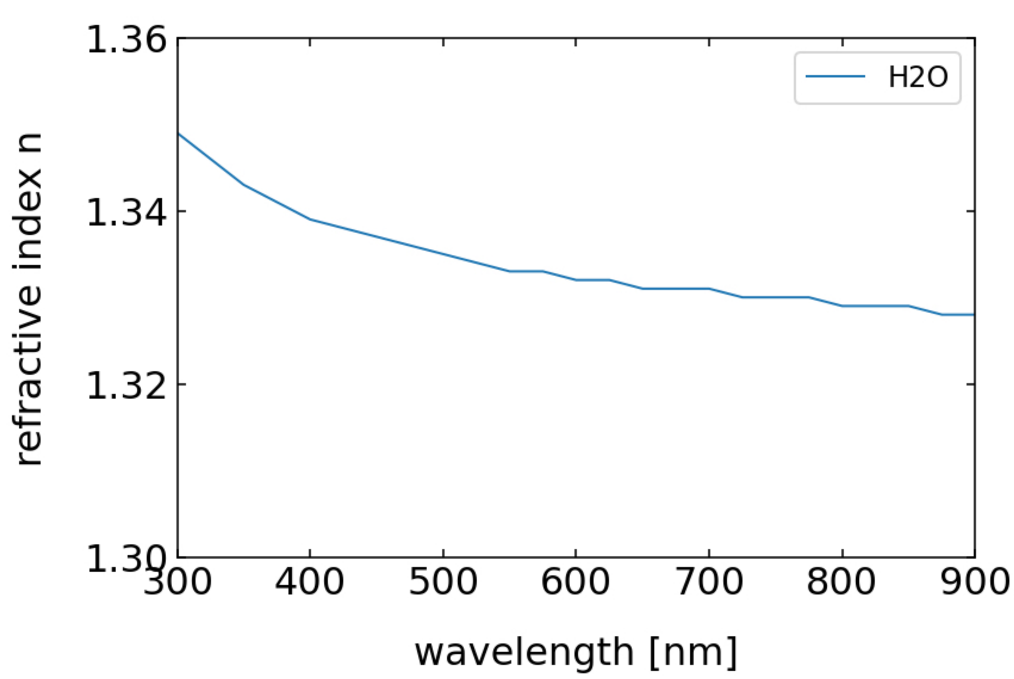
$$n_{air} \sin(\alpha) = n_{water} \sin(\beta)$$

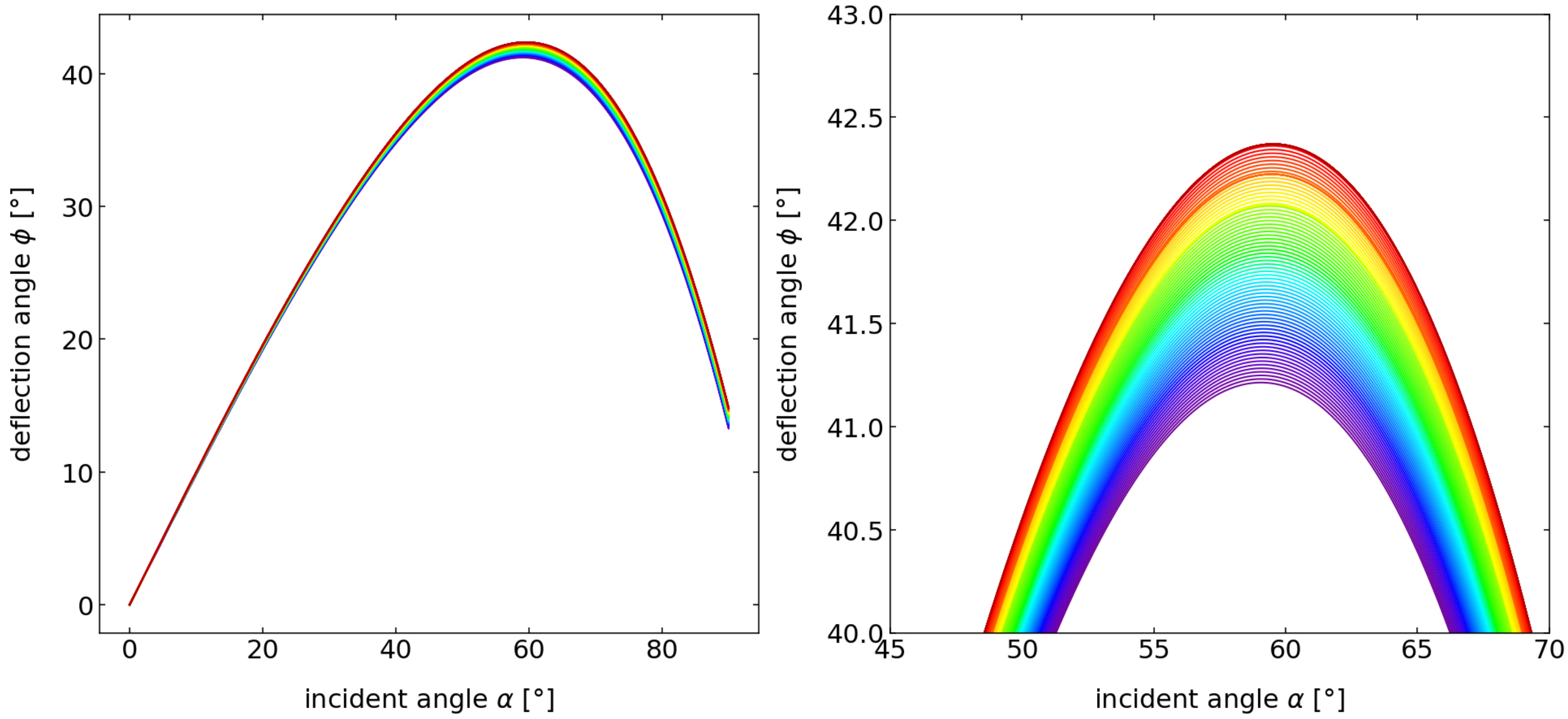
$$\beta = \sin^{-1}\left(\frac{n_{air}}{n_{water}} \sin(\alpha)\right)$$

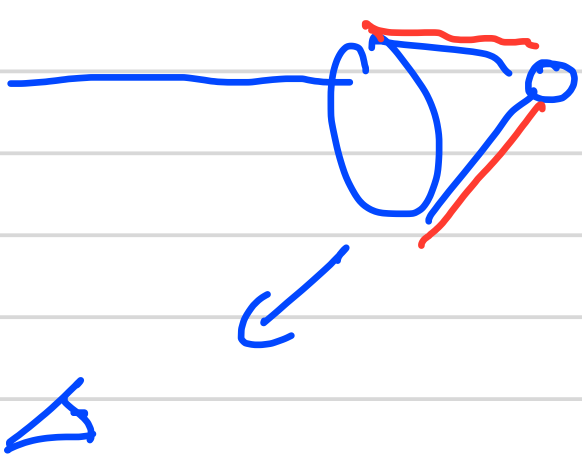
$$\phi = 4 \sin^{-1}\left(\frac{n_{air}}{n_{water}} \sin(\alpha)\right) - 2\alpha$$

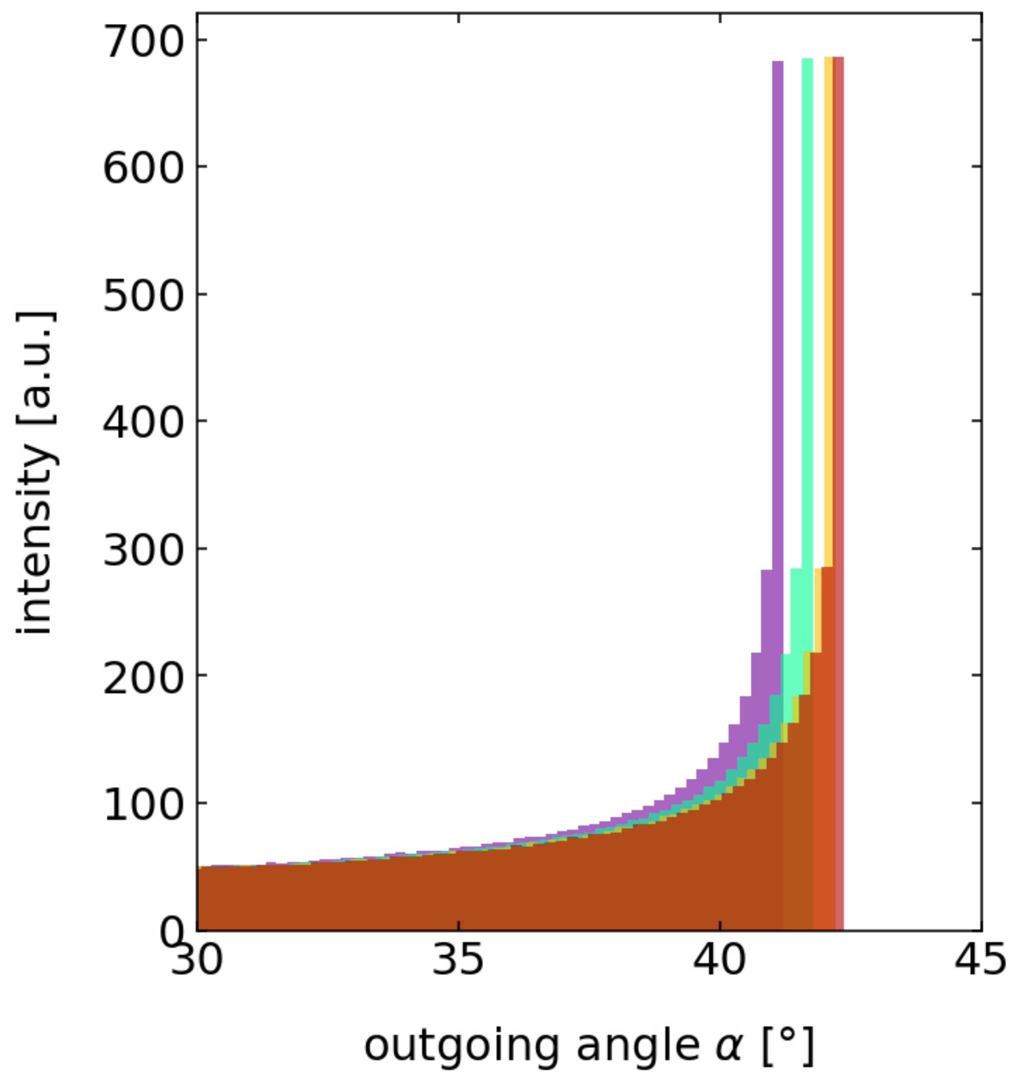


but watch dispersion







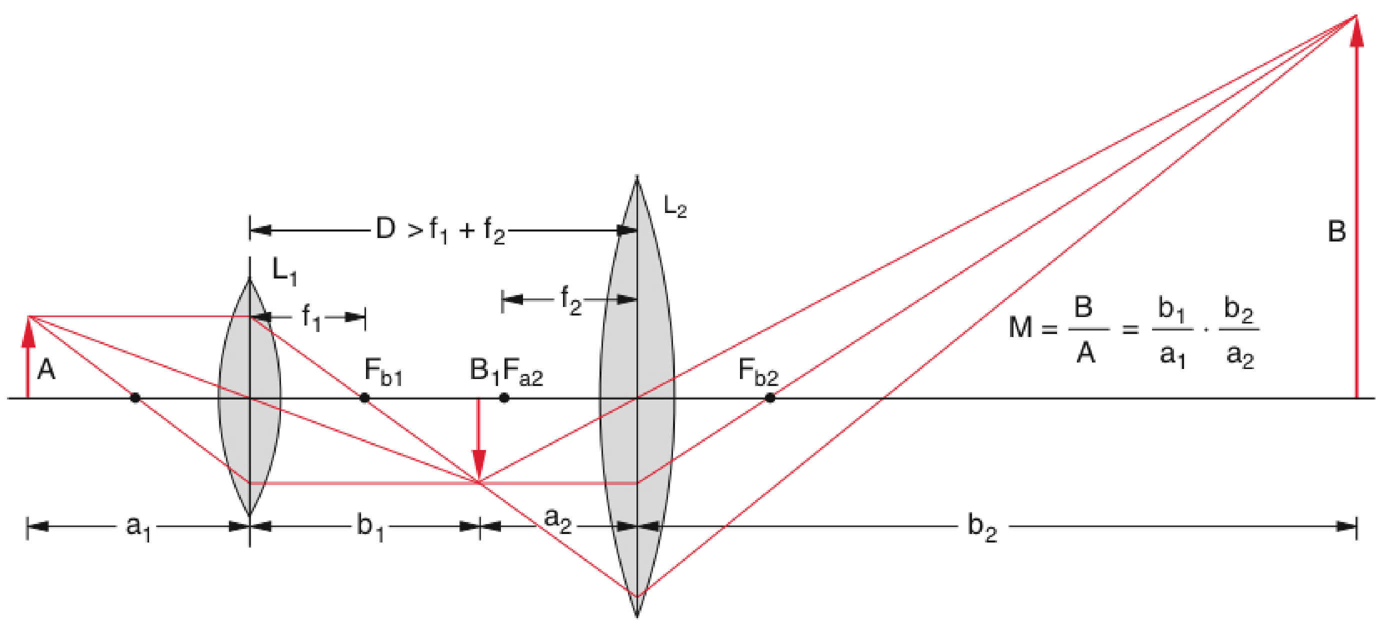


histogram of the inclined
deflection curve

+ yesbaw → bk7

1.4. Lens systems and optical instruments

1.4.1 Lens systems



lens systems are typical for many optical instruments (microscope, telescope)

for example: two biconvex lenses

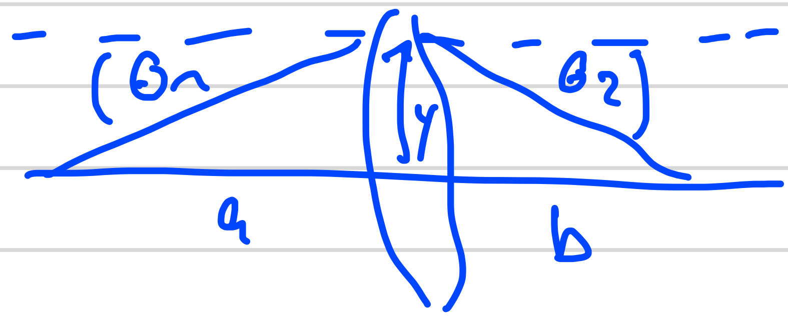
$$\frac{1}{a_1} + \frac{1}{b_1} = \frac{1}{f_1} \rightarrow \text{intermediate image}$$

$$\Rightarrow \frac{1}{a_2} + \frac{1}{b_2} = \frac{1}{f_2} \rightarrow \text{final image}$$

$$b_1 + a_2 = D \Rightarrow \underline{a_2 = D - b_1}$$

more direct method

matrices



$$\theta_2 = \theta_1 - \frac{y_1}{f} \quad , \quad y_2 = y_1$$

$$\rightsquigarrow \begin{bmatrix} y_2 \\ \theta_2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ -\frac{1}{f} & 1 \end{bmatrix} \begin{bmatrix} y_1 \\ \theta_1 \end{bmatrix}$$

$\hookrightarrow$ matrix for a lens

for free space

$$\begin{bmatrix} 1 & D \\ 0 & 1 \end{bmatrix}$$

$\rightsquigarrow$ lens system

lens + free space + lens

$$\begin{bmatrix} y_2 \\ \theta_2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ -\frac{1}{f_2} & 1 \end{bmatrix} \begin{bmatrix} 1 & d \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ -\frac{1}{f_1} & 1 \end{bmatrix}$$

$$\begin{bmatrix} y_2 \\ \theta_2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ -\frac{1}{f_2} & 1 \end{bmatrix} \begin{bmatrix} 1 & D \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ -\frac{1}{f_1} & 1 \end{bmatrix}$$

$$\begin{bmatrix} y_2 \\ \theta_2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ -\frac{1}{f_2} & 1 \end{bmatrix} \begin{bmatrix} 1 - \frac{D}{f_1} & D \\ -\frac{1}{f_1} & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 - \frac{D}{f_1} & D \\ \underbrace{-\frac{1}{f_2}\left(1 - \frac{D}{f_1}\right) - \frac{1}{f_1}}_{-\frac{1}{f}} & 1 - \frac{D}{f_2} \end{bmatrix}$$

$$\boxed{\frac{1}{f} = \frac{1}{f_2} + \frac{1}{f_1} - \frac{D}{f_1 f_2}}$$

effective focal length of two lenses

example $D \ll f_1, f_2$ close lenses

$$\rightsquigarrow \quad \frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2}$$

refractive powers add up

beam expander: $\quad D = f_1 + f_2$

$$\varnothing\, y_1 \longrightarrow \varnothing\, y_2$$

$$\frac{y_2}{y_1} = \frac{f_2}{f_1}$$

$$1 - \frac{D}{f_1} = 1 - \frac{f_1+f_2}{f_1} = 1 - 1 + \frac{f_2}{f_1} = \frac{f_2}{f_1}$$

Magnification:

$$M = M_1 \cdot M_2 = \left(-\frac{b_1}{a_1}\cdot\right)\left(-\frac{b_2}{a_2}\right)$$

$$= \frac{b_1 b_2}{a_1(D - b_1)} \qquad \text{with } a_2 = D - b_2$$

$$= \frac{f_1}{(f_1 - a_1)} \cdot \frac{f_2}{(f_2 + b_1 - D)}$$

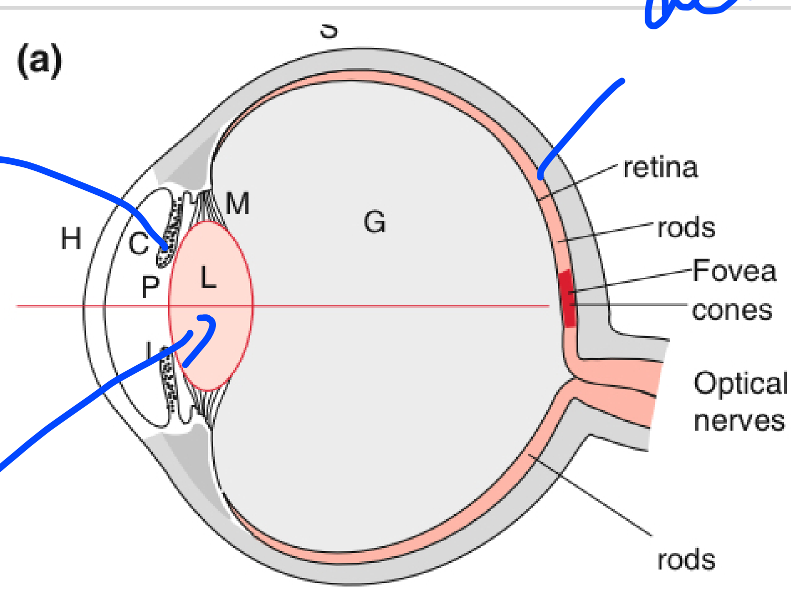
$$\leadsto \boxed{M = \frac{1}{1 - \frac{a_1}{f_1} - \frac{a_1 + D}{f_2} + \frac{a_1 D}{f_1 f_2}}}$$

1.4.2 Optical Instruments

1.4.2.1 The Eye

detector

aperture

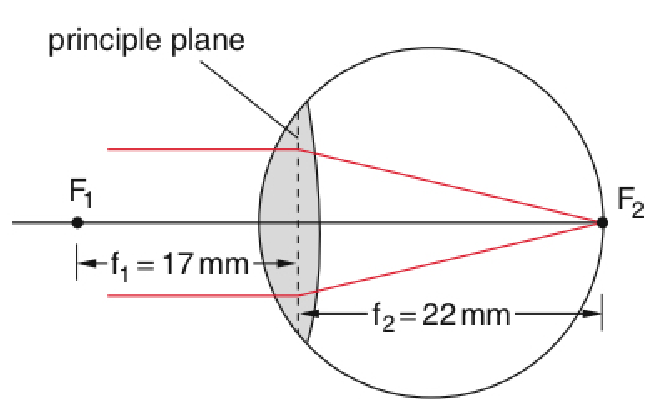


adaptive lens with muscles connected to :

lens

relaxed eye

$f_1 = 17$, $f_2 = 22$ mm



close range

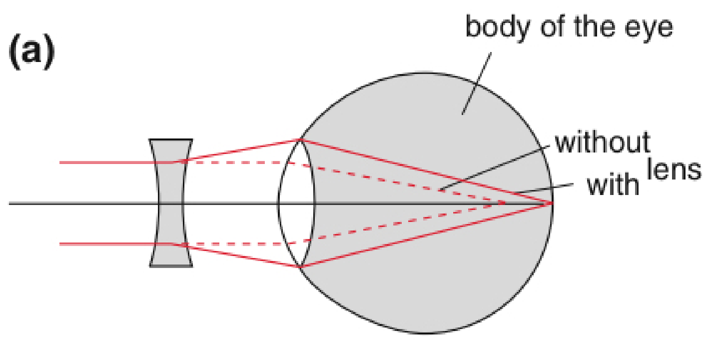
$f_1 = 14$ mm

$f_2 = 18$ mm

shot sightedness

distant objects create a focus too short to be sharp on retina

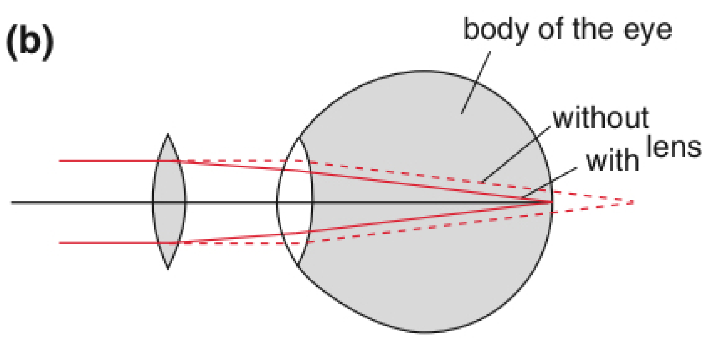
(a)



far sightedness

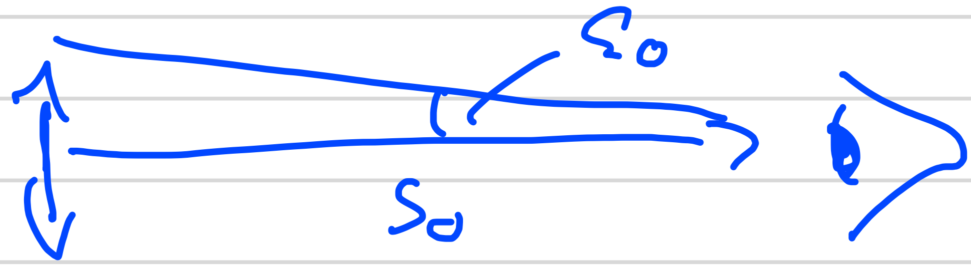
focus too long, correct with convex lens

(b)



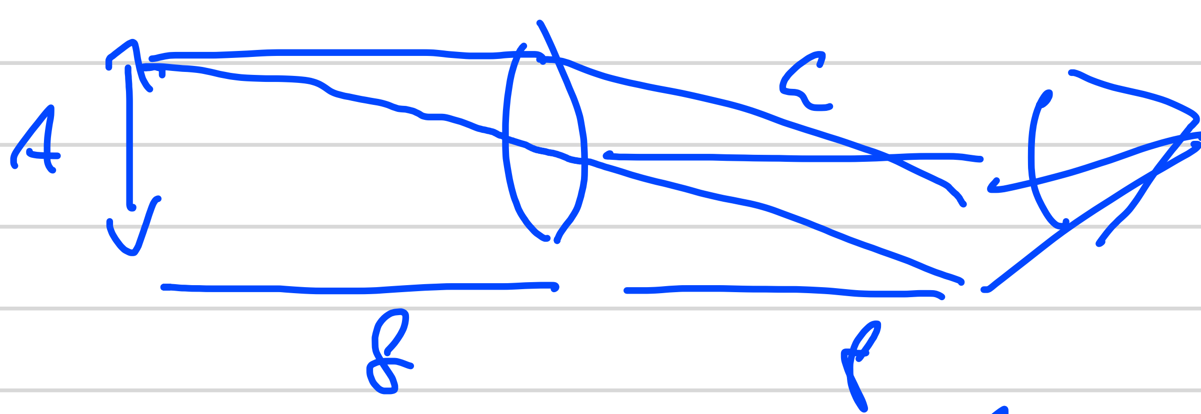
1.4.2.2. Magnifying lens

magnifying instruments typically increase the visual angle



$\tan \varepsilon_0 = \frac{A}{s_0}$ $\qquad$ $\varepsilon_0 \approx \frac{A}{s_0}$

$s_0 \approx 25\ \text{cm}$ $\qquad$ clear visual range



object app. at infinite distance

$\tan \varepsilon = \frac{A}{f}$ $\qquad$ $\varepsilon \approx \frac{A}{f}$

$$V = \frac{\tan \varepsilon}{\tan \varepsilon_0} = \frac{A}{f} \cdot \frac{s_0}{A} = \frac{s_0}{f}$$

angular magnification

$\rightsquigarrow$ lens brings object much closer from s_0 to the eye

<u>with virtual image</u>

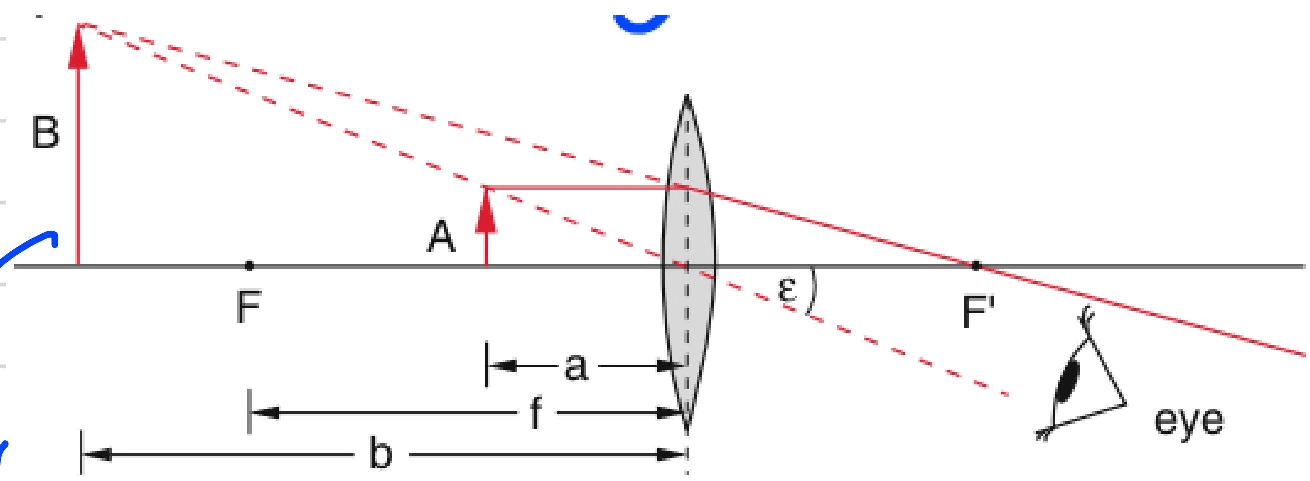


image appears at finite distance

$$V_L = \frac{\tan \varepsilon}{\tan \varepsilon_0} = \frac{B/b}{A/s_0} = \frac{A/a}{A/s_0} = \frac{s_0}{a}$$

with $$\frac{1}{f} = \frac{1}{a} - \frac{1}{b}$$

$$\rightsquigarrow \frac{1}{a} = \frac{b-f}{b \cdot f}$$

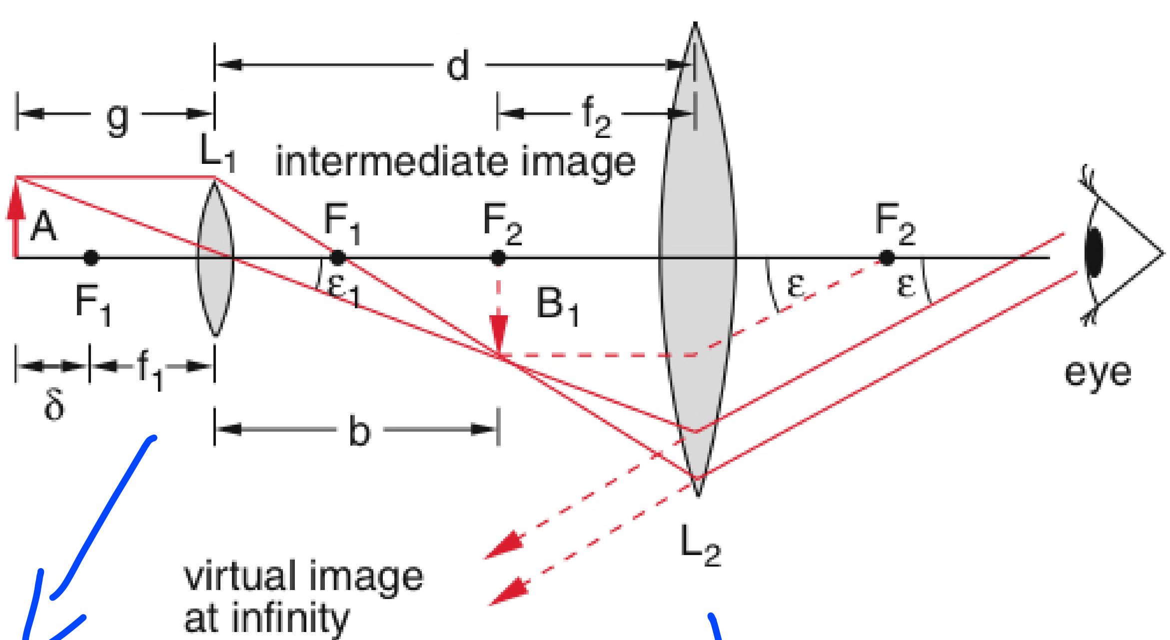
$$\rightsquigarrow V_L = \frac{s_0 + f}{f} = \frac{s_0}{f} + 1$$

magnifying glasses often used in form of eye pieces for microscopes

Microscope [late 1600 Leeuwenhoek]

objective lens

eye piece



real inverted intermediate image

virtual image of the intermediate image

$$\frac{1}{f_1} = \frac{1}{g} + \frac{1}{b}$$

$$b = \frac{g \cdot f_1}{g - f_1} = \frac{g f_1}{\delta}$$

$$\delta \to 0 \quad b \gg g$$

magnifying glass (eye piece)

with the $\varepsilon = \frac{B_1}{f_1} \approx \frac{G b}{g f_2}$

without the $\varepsilon_0 = \frac{G}{s_0}$

$\rightsquigarrow$ $V_m = \frac{G b s_0}{G b f_2} = \frac{b s_0}{g f_2}$

lens distance $d = b + f_2$, $g \approx f_1$

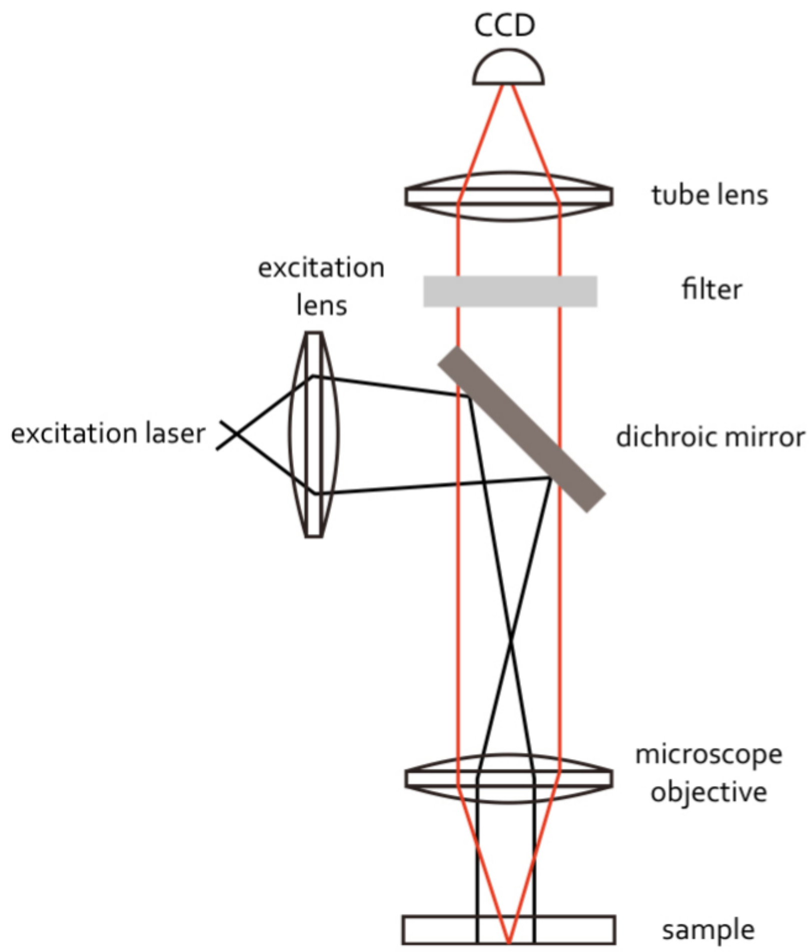
$$V_m = \frac{(d - f_2) s_0}{f_1 f_2}$$

magnification chose by f_1, f_2

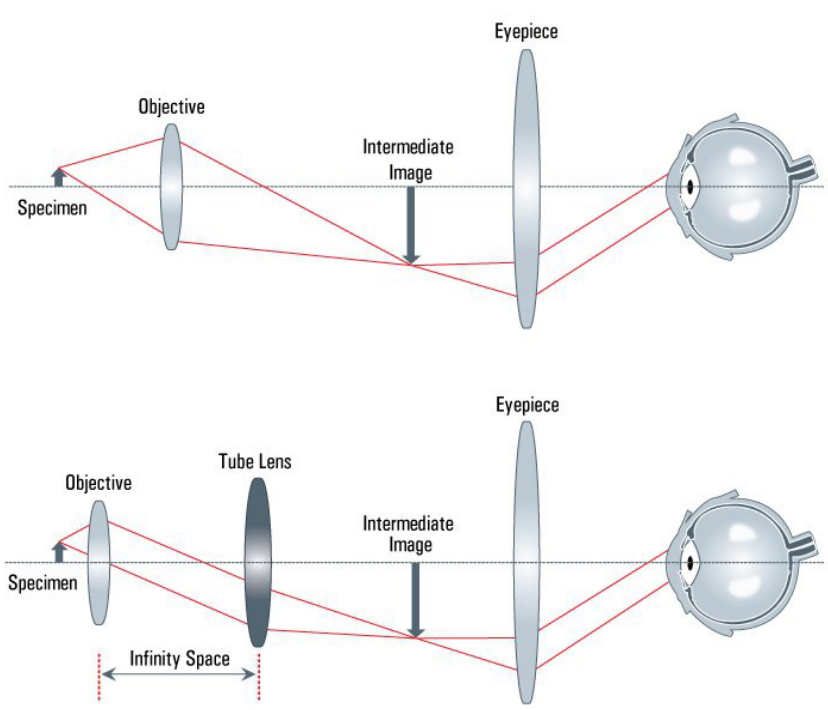
modern objectives

infinity corrected

objective + tube lens define



tube lens + objective define the magnification

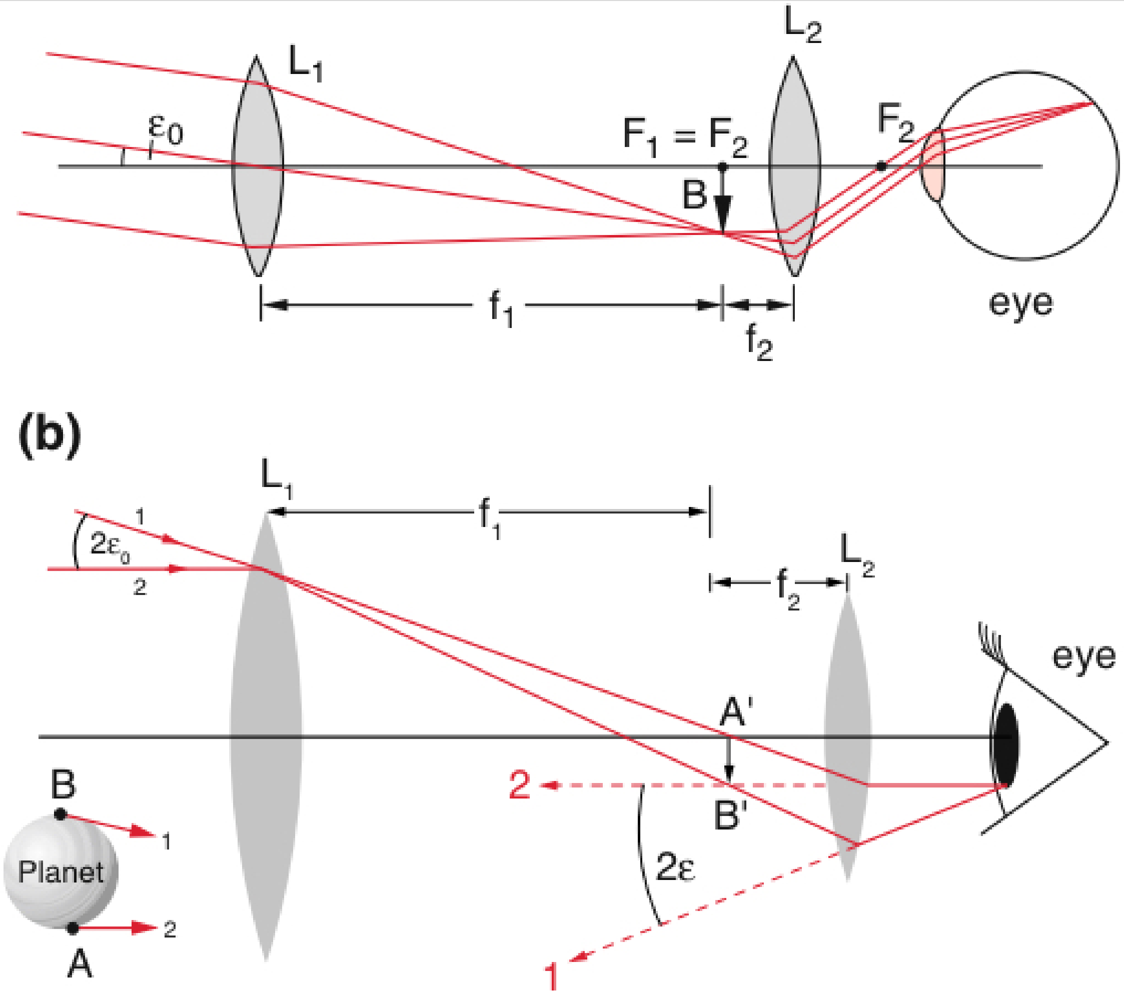


normal vs. infinity optics

Telescopes

Kepler Telescope

L1 has large focal length



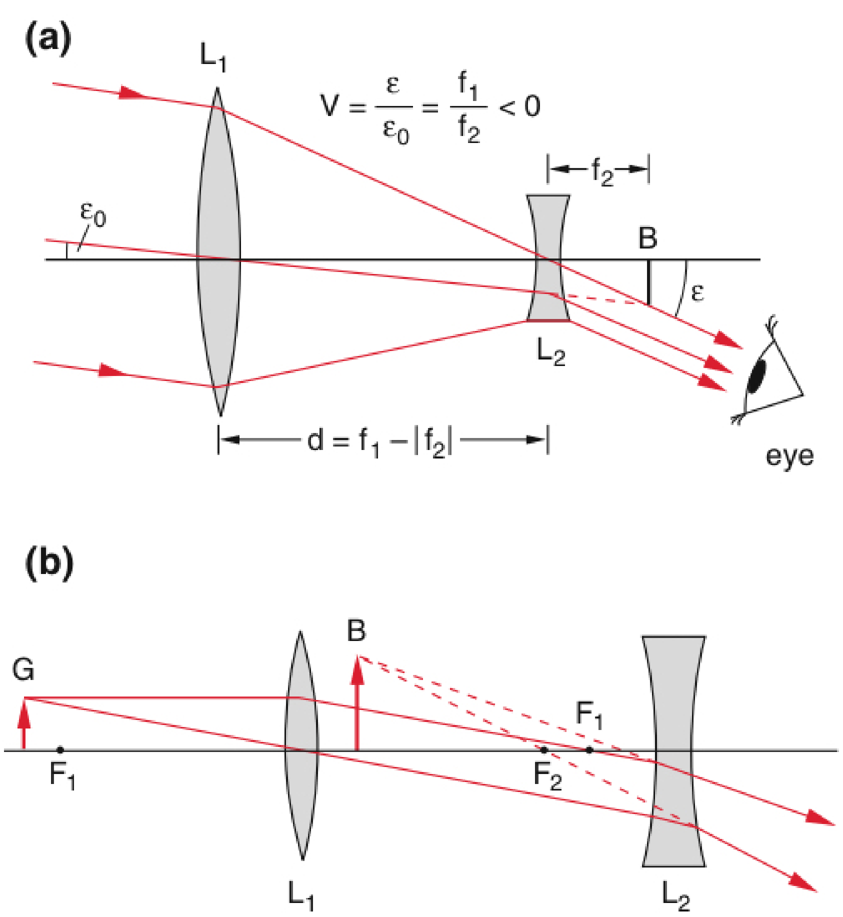
L2 magnifying glass

planet diameter D   observer

$\rightsquigarrow 2\varepsilon_0 = \frac{D}{f_1}$ $2\varepsilon = \frac{D}{f_2}$

$$V_F = \frac{\varepsilon}{\varepsilon_0} = \frac{D}{f_2} \cdot \frac{f_1}{D} = \frac{f_1}{f_2}$$

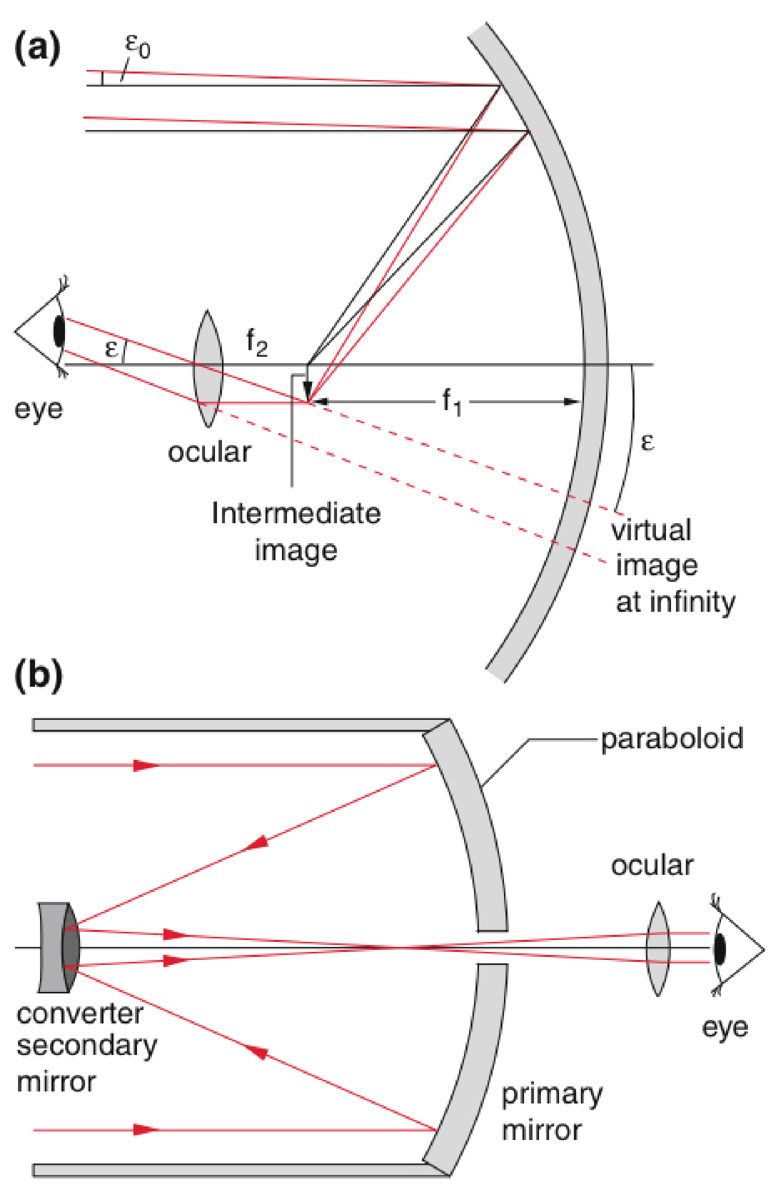
→ image of the Kepler telescope is inverted



upright image telescope

a Galilei telescope or terrestrial

L2 is a diverging lens

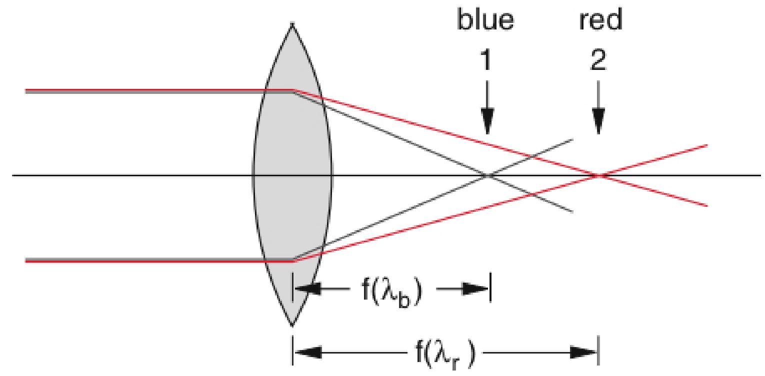


reflecting telescopes

Aberrations

whereas the paraxial appr. is not anymore valid or $n(\lambda)$

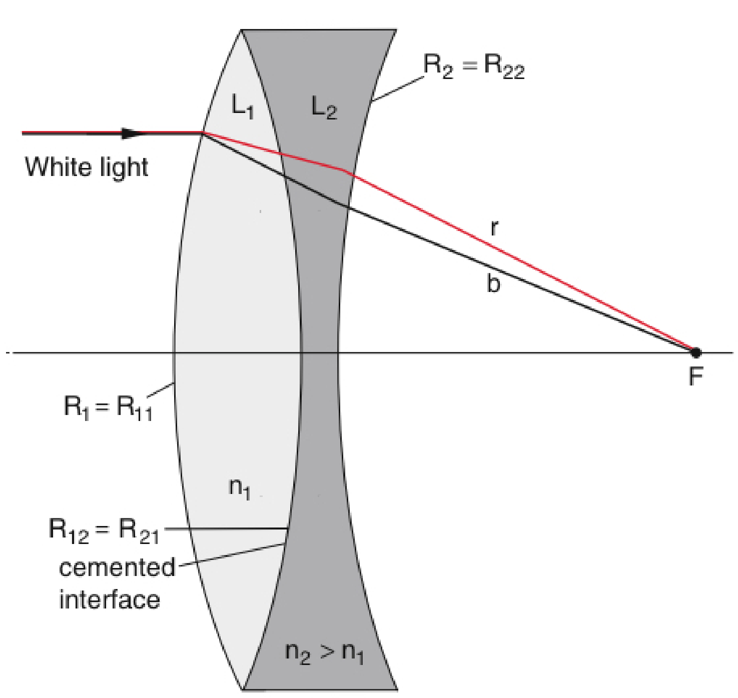
Chromatic aberration



dispersion curve again

- is due to the wavelength dispersion of the refraction index

- can be corrected with a lens pair with $n_1(\lambda)$ and $n_2(\lambda)$



$$\frac{R_2}{R_1} = \frac{n_{2b} - n_{2r}}{2(n_{1b} - n_{1r}) - (n_{2b} - n_{2r})}$$

$$\frac{1}{f_i} = (n_i - 1)\,\rho_i$$

with $\rho_i = \dfrac{(R_{i2} - R_{i1})}{R_{i2} \cdot R_{i1}}$

$\rightarrow$ focal length is halved (if lens are close)

$$\frac{1}{f} = (n_1 - 1)\rho_1 - (n_2 - 1)\rho_2$$

if the focal length of blue and red light is the same then

$$(n_{1r} - 1)\rho_1 + (n_{2r} - 1)\rho_2$$

$$= (n_{1b} - 1)\rho_1 + (n_{2b} - 1)\rho_2$$

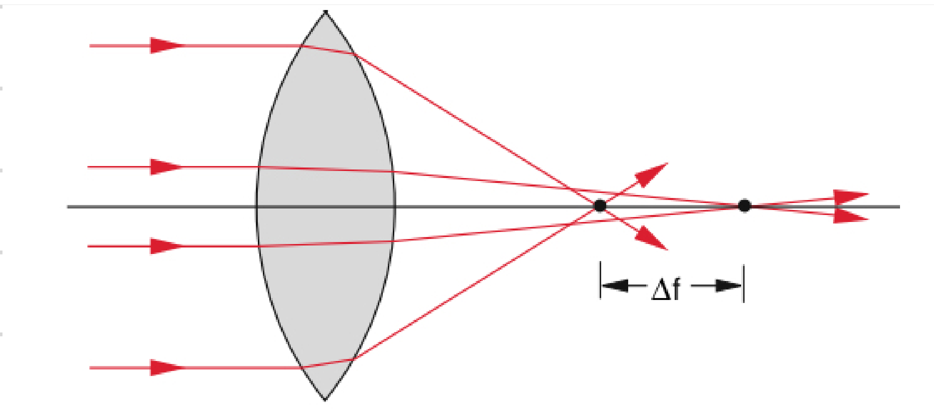
$$\Rightarrow \frac{\rho_1}{\rho_2} = -\frac{n_{2b} - n_{2r}}{n_{1b} - n_{1r}}$$

for cemented lenses $R_{12} = R_{21}$

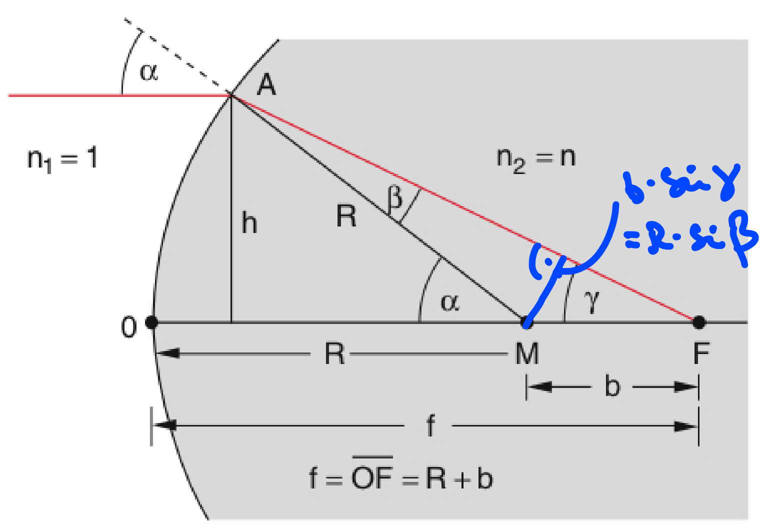
first lens symmetric

$$R_{11} = R_1 = -R_{12} = -R_{21}, \quad R_{22} = R_2$$

Spherical Aberration



non-paraxial rays have different focal length



$$f = R + b \,, \quad b = R \cdot \frac{\sin\beta}{\sin\gamma}$$

$$f = R \cdot \frac{h}{n \sin(\gamma)}$$

$$f = R\left[1 + \frac{1}{n\cos\beta - \cos\alpha}\right]$$

$\sin\beta = \sin\alpha / n \,, \quad \sin\alpha = \frac{h}{R}$

$\alpha = \beta + \gamma$

$$f = R\left[1 + \frac{1}{n\sqrt{1 - \frac{h^2}{n^2R^2}} - \sqrt{1 - \frac{h^2}{R^2}}}\right]$$

focal distance   decreases with h

$$f = R\left[\frac{n}{n-1} - \frac{h^2}{2n(n-1)R^2}\right]$$

imaging equation

$$\frac{1}{a} + \frac{n}{b} = \frac{n-1}{R} + h^2\left[\frac{1}{2a}\left(\frac{1}{a}+\frac{1}{R}\right)^2 + \frac{n}{2b}\left(\frac{1}{R}-\frac{1}{b}\right)^2\right]$$

similarly obtained for a thin lens

$$\sin\alpha \approx \alpha - \frac{1}{3!}\alpha^3$$

$$\cos\alpha \approx 1 - \frac{\alpha^2}{2}$$

- correction by

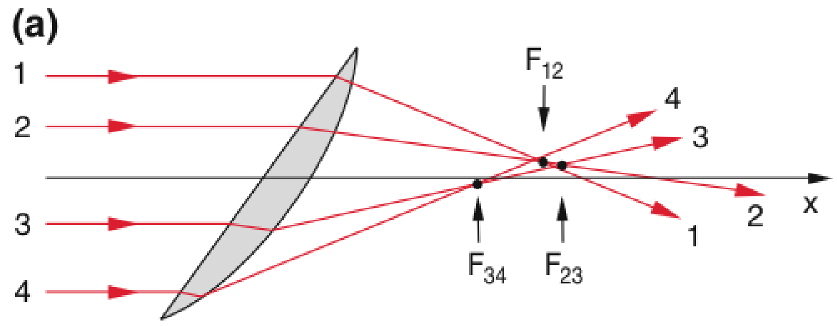
  $\rightarrow$ suppression of far rays

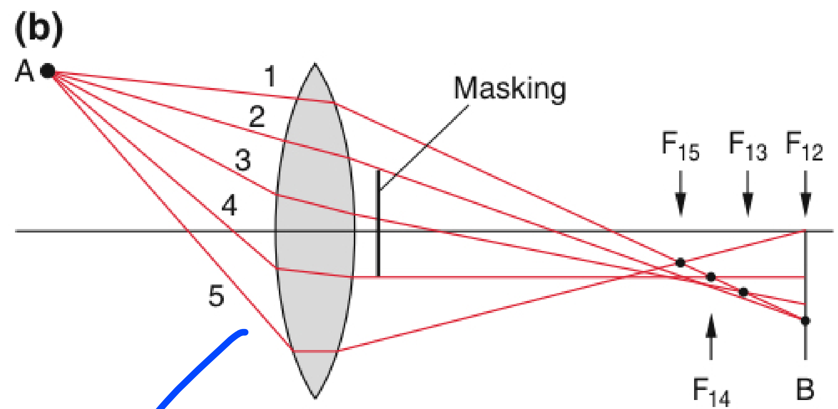
  $\rightarrow$ plano-convex lenses with convex to object

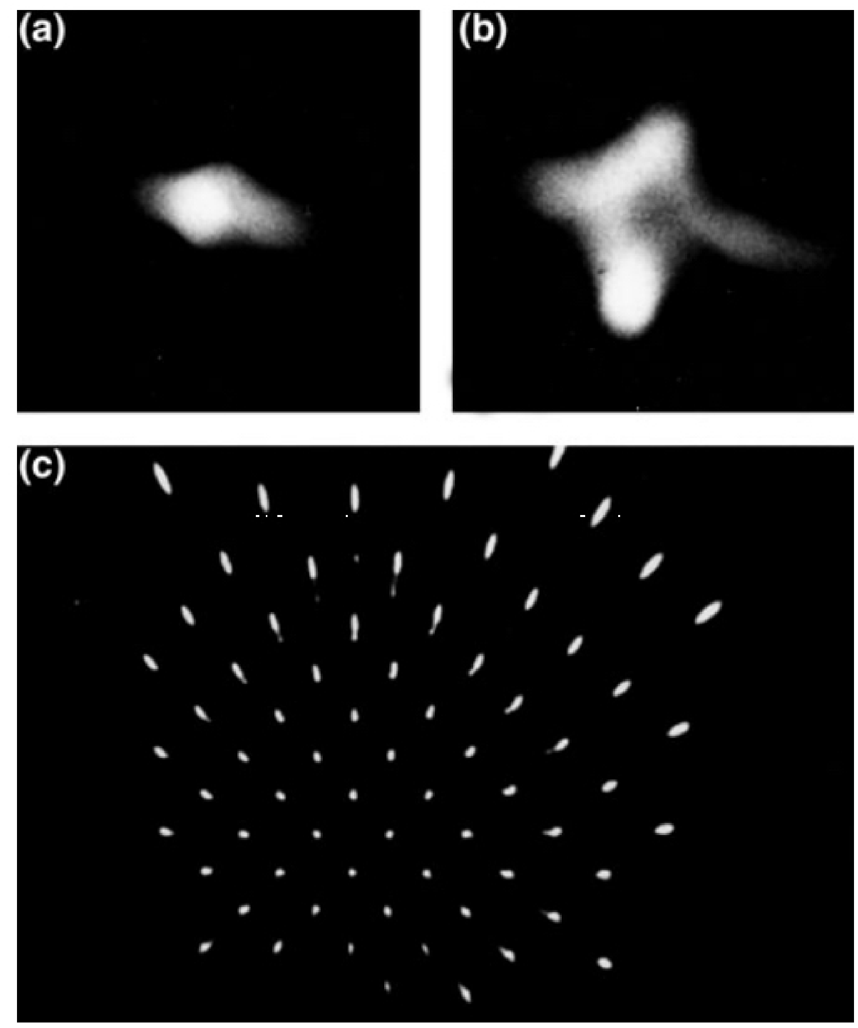
  $\rightarrow$ several convex concave lenses

  $\rightarrow$ non spherical lenses

Coma

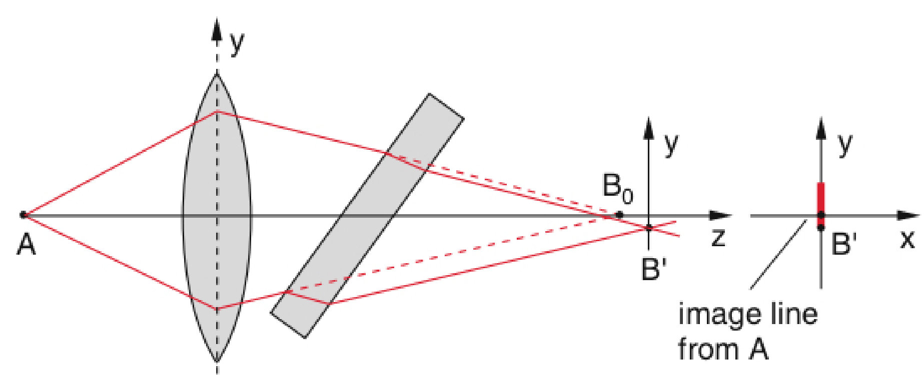


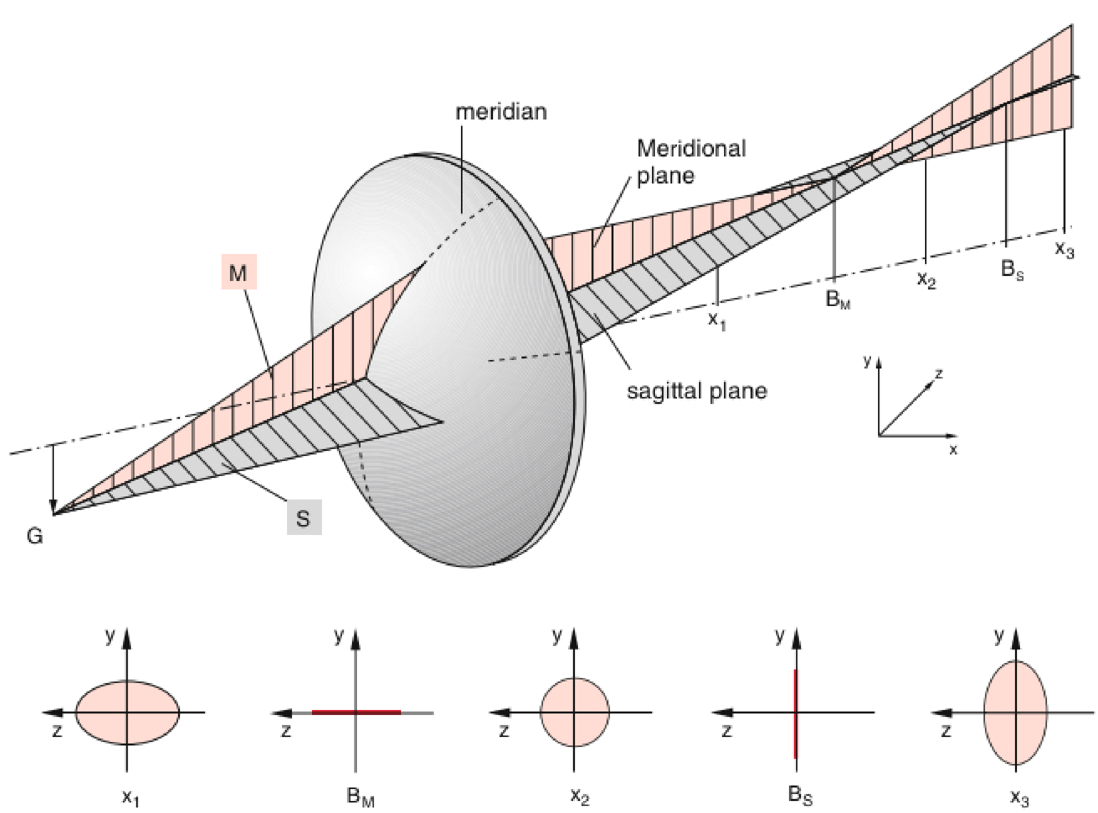




no small angle approx

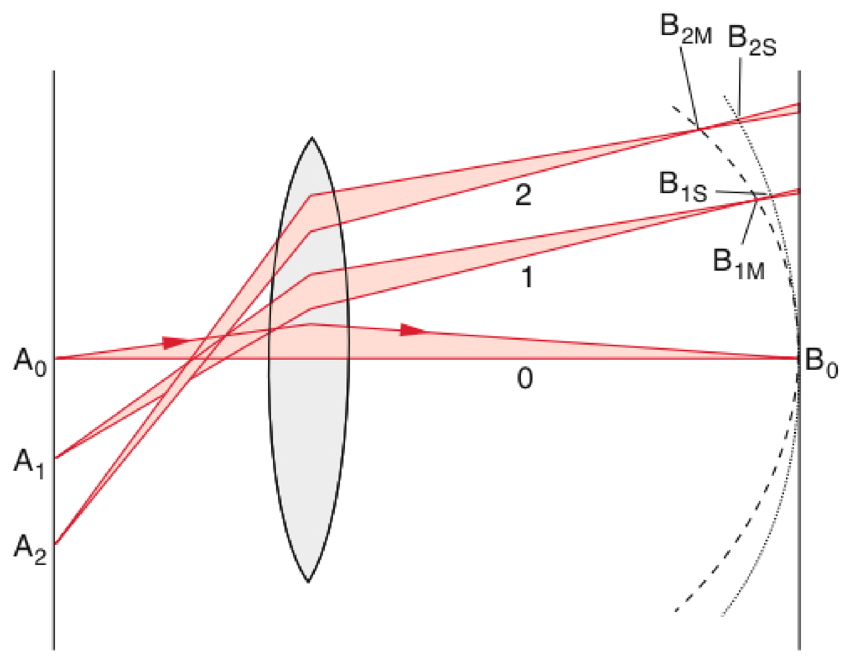
Astigmatism



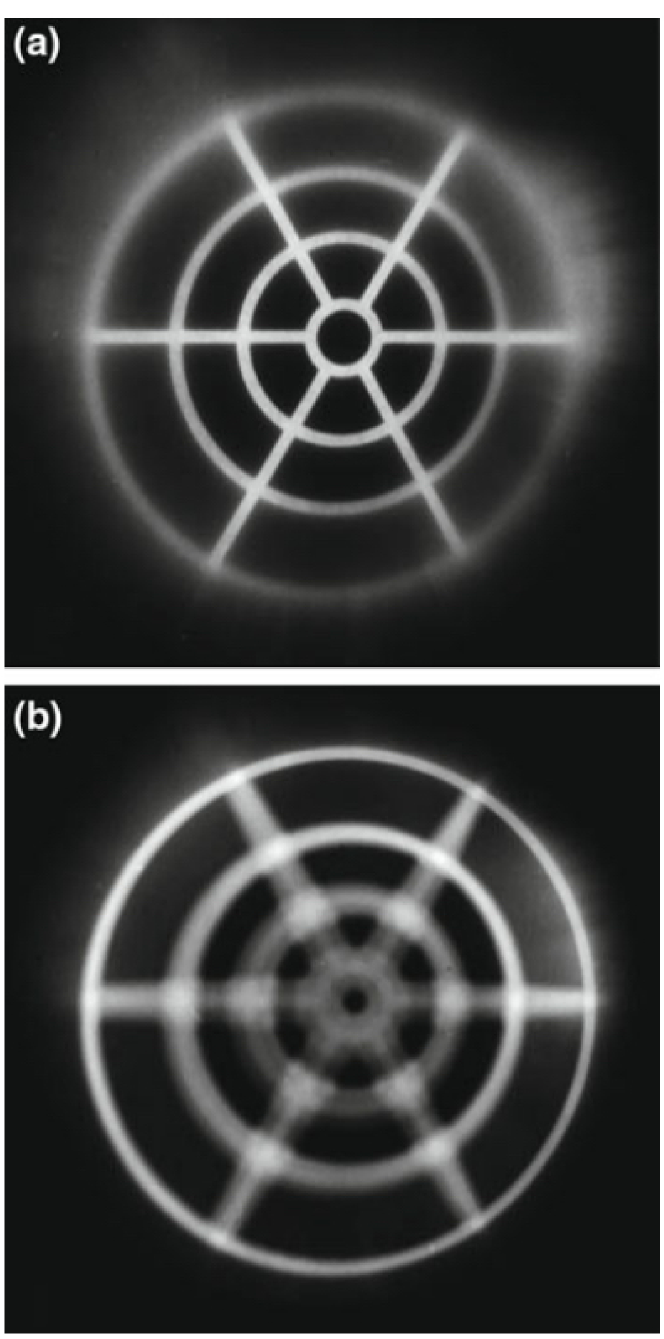


the path
through
the lens is

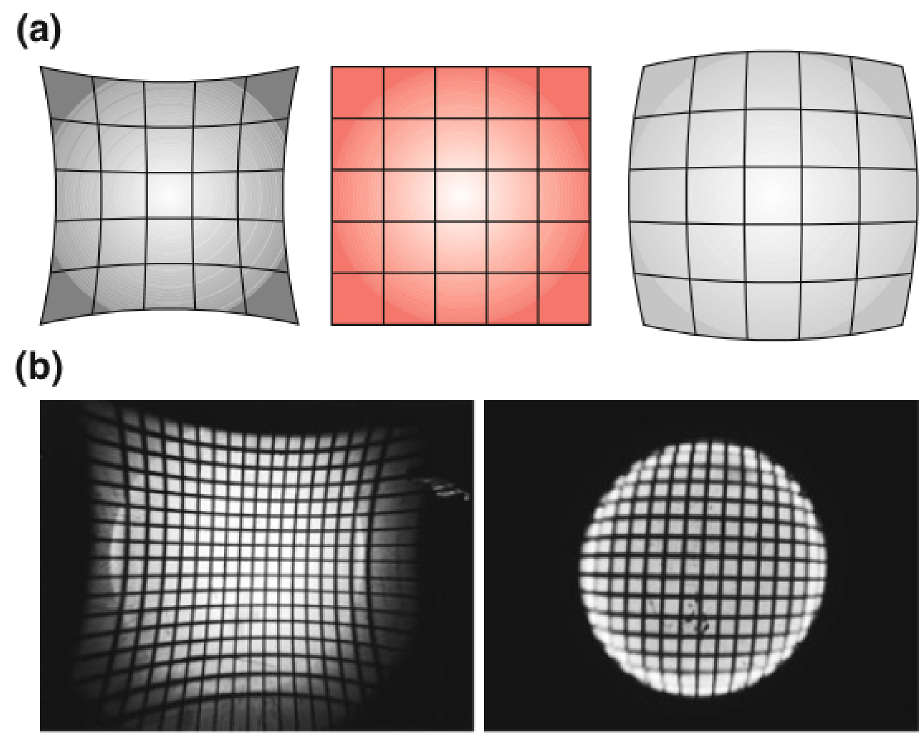
Field distortion

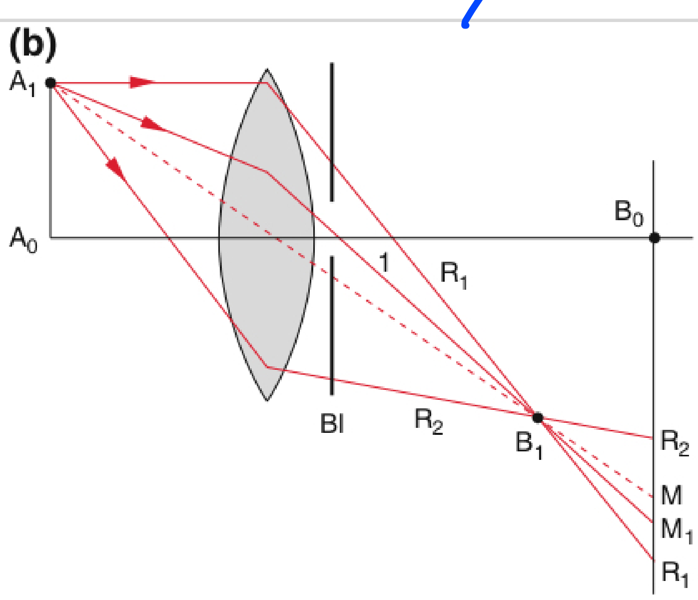


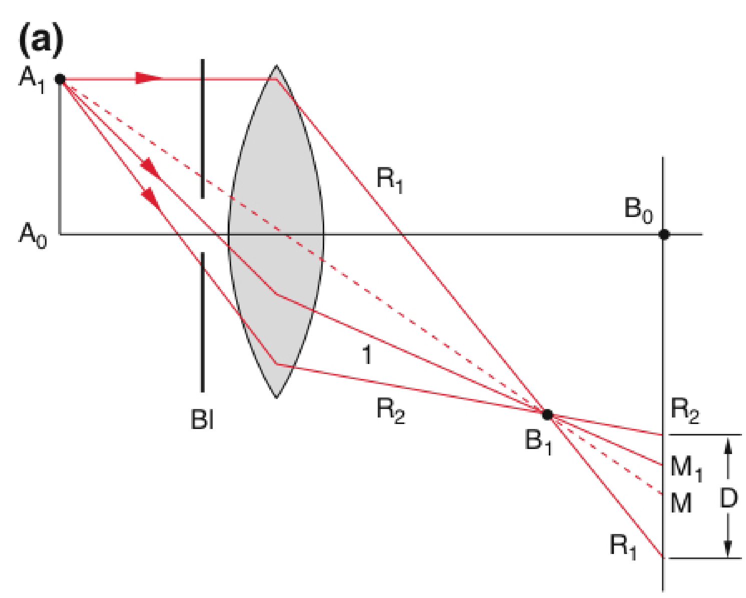
conjugated plane of the object plane is not a plane



field curvature







Shift of R_1 becomes larger with distance h

shift of R_1 gets smaller with h

2. Wave Optics

- signatures of wave optics
  - -> interference
  - -> diffraction
  - -> color

- show slides on diffraction and interference
- show color

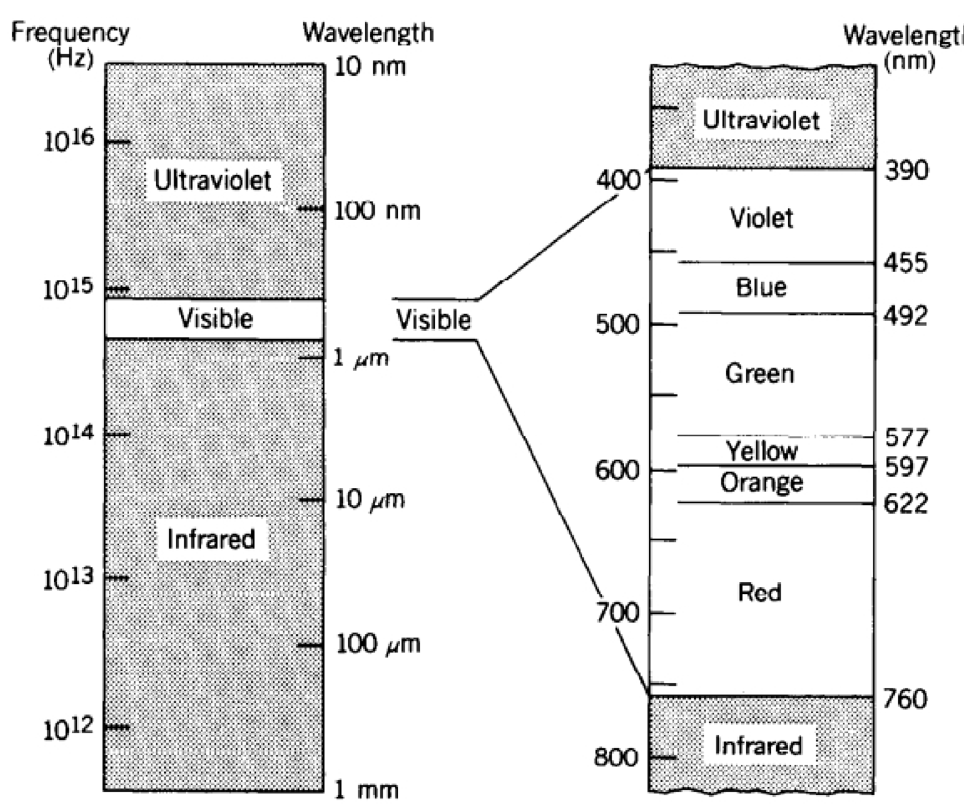


Figure 2.0-1 Optical frequencies and wavelengths.

Postulates of wave optics

- $c = \frac{c_0}{n}$ speed of light in a medium

- light is described by waves

  → a wave is a periodic function of a physical quantity in space and time

- → transport of energy → $|\text{amplitude}|^2$

2.1. wave equation

$$\nabla^2 u - \frac{1}{c^2}\frac{\partial^2 u}{\partial t^2} = 0$$

$$\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$$

Laplace operator

→ linear operator → superposition

u_1, u_2 solution → $u_1 + u_2$

e.g.

$$u = a_1 u_1(\vec{r},t) + a_2 u_2(\vec{r},t)$$

$$\nabla^2 u - \frac{1}{c^2}\frac{\partial^2 u}{\partial t^2} = 0$$

$$a_1 \nabla^2 u_1 - \frac{a_1}{c^2}\frac{\partial^2 u_1}{\partial t^2} + a_2 \nabla^2 u_2 - \frac{a_2}{c^2}\frac{\partial^2 u_2}{\partial t^2}$$

$$= 0$$

$$\rightsquigarrow \underbrace{a_1 \nabla^2 u_1 - \frac{a_1}{c_2}\frac{\partial^2 u_1}{\partial t^2}}\ = - \underbrace{a_2 \nabla^2 u_2 - \frac{a_2}{c^2}\frac{\partial^2 u_2}{\partial t^2}}$$

both sides are indep

$$\rightsquigarrow a_1 \nabla^2 u_1 - \frac{a_1}{c^2}\frac{\partial^2 u_1}{\partial t^2} = 0 = a_2 \nabla^2 u_2 \ldots$$

→ superposition is allowed and this is making wave optics + ...

intensity of waves

Intensity

$$I(\vec{r},t) = 2\langle u^2(\vec{r},t)\rangle$$

units are $\frac{W}{m^2}$

- $\langle .. \rangle$ average over an optical cycle

i.e. 600 nm light
cycle $2 \cdot 10^{-15}\,s = 2\,fs$

Power

$$P = \int_A I(\vec{r},t)\,dA$$

Monochromatic Wave

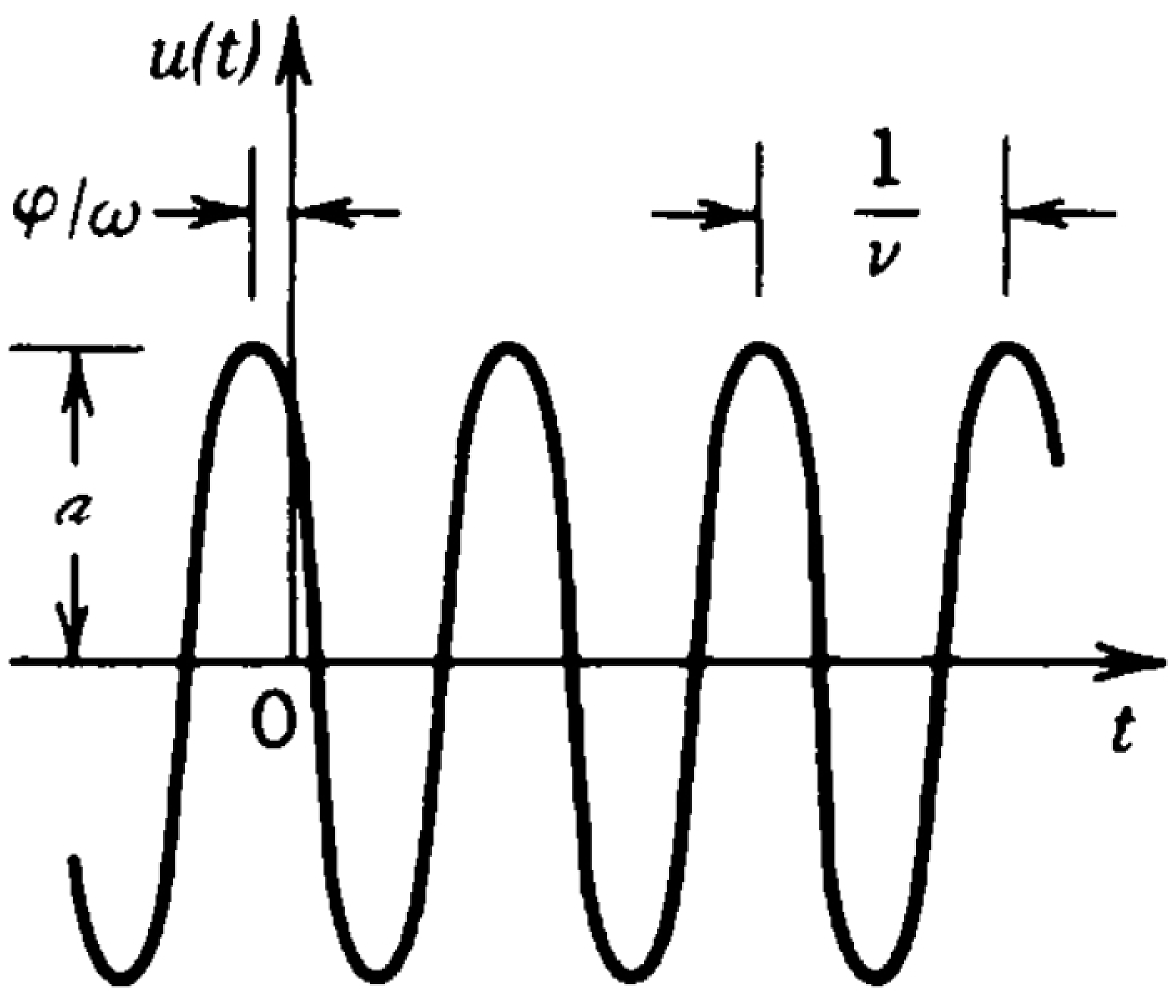
$$u(\vec{r},t) = a(\vec{r})\cos(\omega t + \varphi(\vec{r}))$$

$a(\vec{r})$ = amplitude

$\varphi(\vec{r})$ = phase

$\omega = 2\pi\upsilon$ frequency

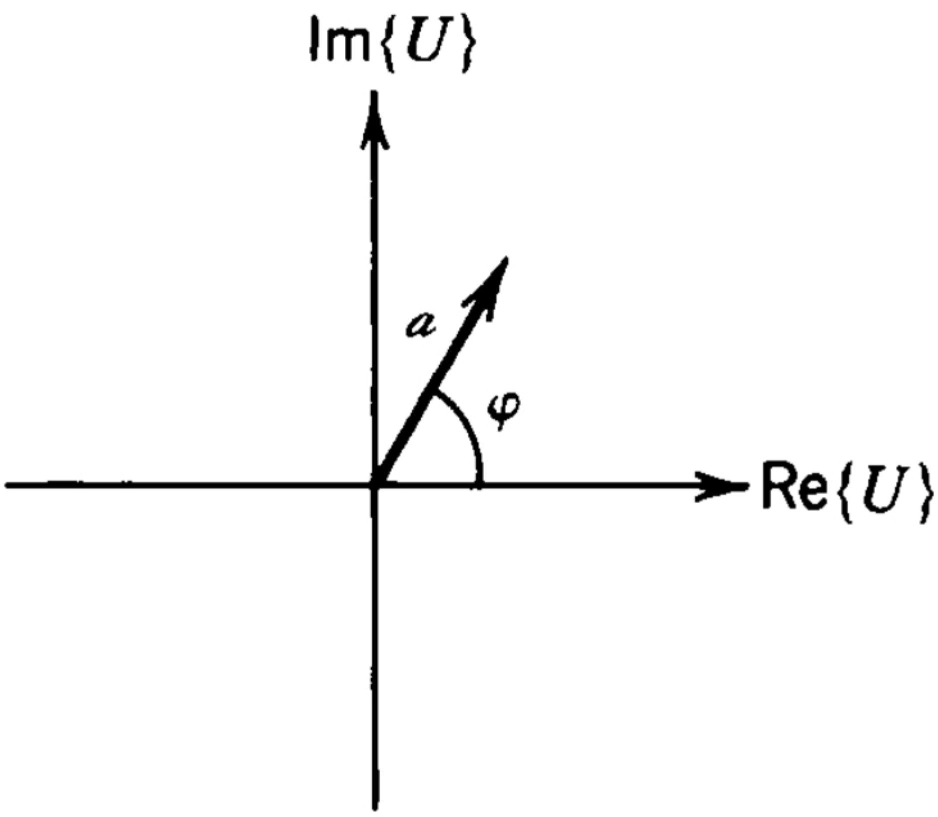
Solution is a harmonic function at the frequency ν at all $\vec{r}$



Complex representation

$$U(\vec{r},t) = a(\vec{r})\cdot e^{i\varphi(r)}\cdot e^{i\omega t}$$

$$\rightsquigarrow \quad u(\vec{r},t) = \mathrm{Re}\left\{U(\vec{r},t)\right\} = \frac{1}{2}\left[U(\vec{r},t) + U^{*}(\vec{r},t)\right]$$



Im{U}
a
φ
Re{U}
Im{U(t)}
ω
a
φ
Re{U(t)}

$U(\vec{r},t)$ complex wavefunction

real $u(\vec{r},t)$ is just the real part

$$\sim \boxed{\nabla^2 U - \frac{1}{c^2}\frac{\partial^2 U}{\partial t^2} = 0}$$ wave equation

Complex Amplitude

$$U(\vec{r},t) = U(\vec{r}) \cdot e^{i\omega t}$$
$$U(\vec{r}) = a(\vec{r}) \cdot e^{i\varphi(\vec{r})}$$

$\rightarrow$ Intensity $\sim$ $\underline{\underline{I(\vec{r}) = |U(r)|^2}}$

Wavefronts

surfaces of $\varphi(\vec{r}) = \text{const}$

$\varphi(\vec{r}) = 2\pi \cdot q$ $\quad q \ldots$ integer wavefront normal

$$\vec{n} = \left\{\frac{\partial\varphi}{\partial x}, \frac{\partial\varphi}{\partial y}, \frac{\partial\varphi}{\partial z}\right\}$$

2.1.1 <u>Plane wave</u>

$$U(\vec{r}) = A \cdot e^{-i\vec{k}\cdot\vec{r}}$$

↳ complex

$$\vec{k} = \{k_x, k_y, k_z\}$$ wave vector

$$\sqrt{k^2} = \sqrt{k_x^2 + k_y^2 + k_z^2}$$ wave number

phase $\arg\{U(\vec{r})\} = \arg(A) - \vec{k}\vec{r}$

⤷ $\vec{k}\cdot\vec{r} = 2\pi q + \arg(A) \rightarrow$ planes

planes are separated by λ

⤷ $k = \frac{2\pi}{\lambda}$ $\lambda = \frac{2\pi}{k}$

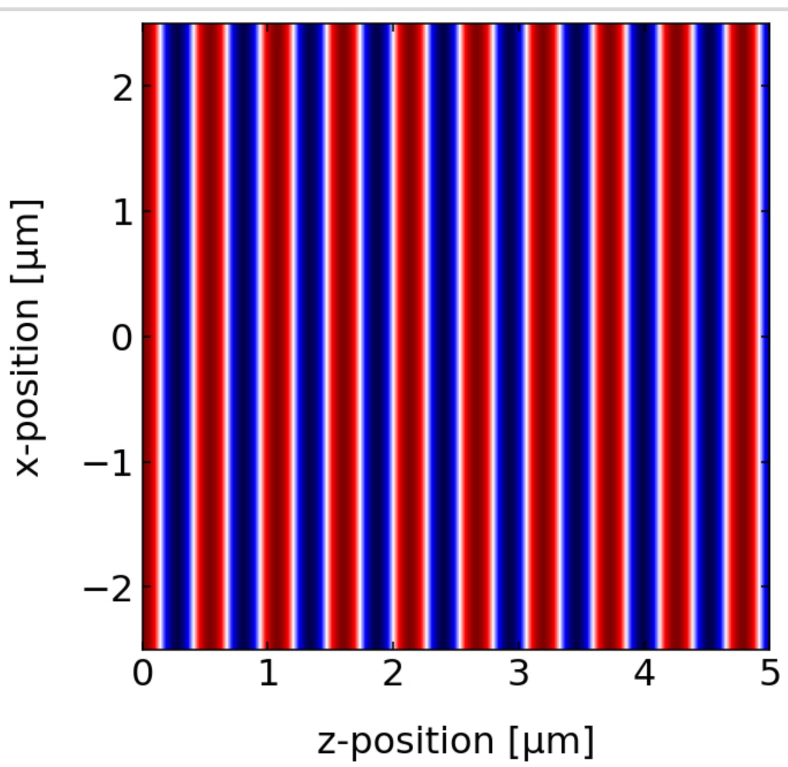
⤷ $$\boxed{\lambda = \frac{c}{\nu}}$$ wavelength

$$I = |A|^2$$

$$u(\vec{r},t) = |A| \cos\left(\omega t - kz + \arg(A)\right]$$

→ wave function is periodic in time with $\frac{2\pi}{\omega}$

→ wave function is periodic in space with $\frac{2\pi}{k}$



plane wave in medium

$$c = \frac{c_0}{n} \qquad \underline{k = n k_0}$$

$$\lambda = \frac{\lambda_0}{n}$$

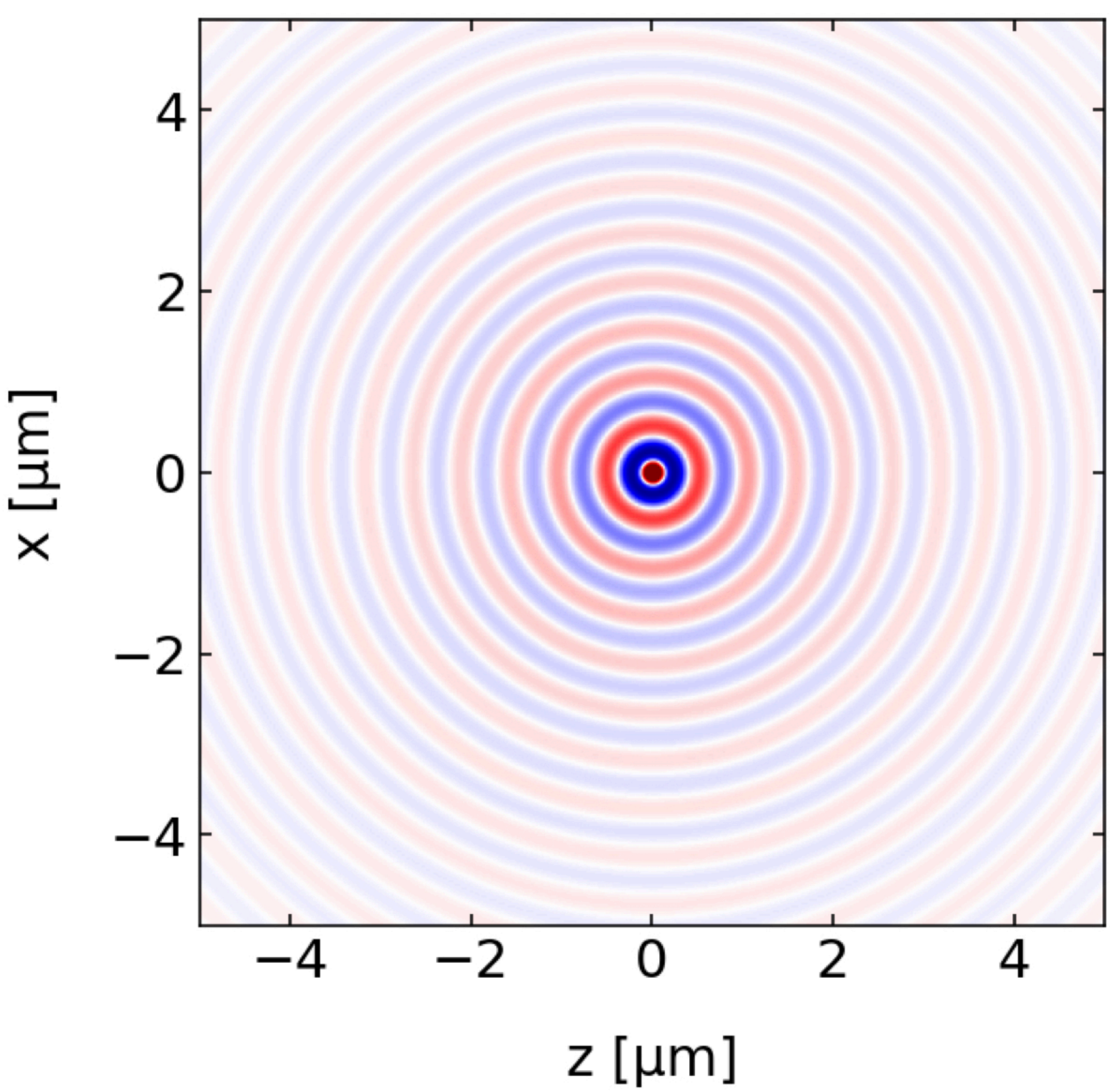
2.1.2 Spherical Wave

$$U(\vec{r}) = \frac{A}{r} \cdot e^{-ikr}$$

$$I(\vec{r}) = \frac{|A|^2}{r^2}$$

with $\arg\{A\} = 0 \;\forall\; kr = 2\pi q$

spheres with $r = q \cdot \lambda$



2.1.3 Interference of two waves

$$U(\vec{r}) = U_1(\vec{r}) + U_2(\vec{r})$$

$$I_1 = |U_1|^2 \qquad I_2 = |U_2|^2$$

$$I = |U|^2 = |U_1 + U_2|^2$$
$$= |U_1|^2 + |U_2|^2$$
$$+ U_1^* U_2 + U_1 U_2^*$$

with

$$U_1 = \sqrt{I_1}\, e^{i\varphi_1}, \quad U_2 = \sqrt{I_2}\, e^{i\varphi_2}$$

$$I = I_1 + I_2 + 2\sqrt{I_1 I_2} \cos\varphi$$

$$\varphi = \varphi_2 - \varphi_1$$

→ intensity is not the sum of the two intensities

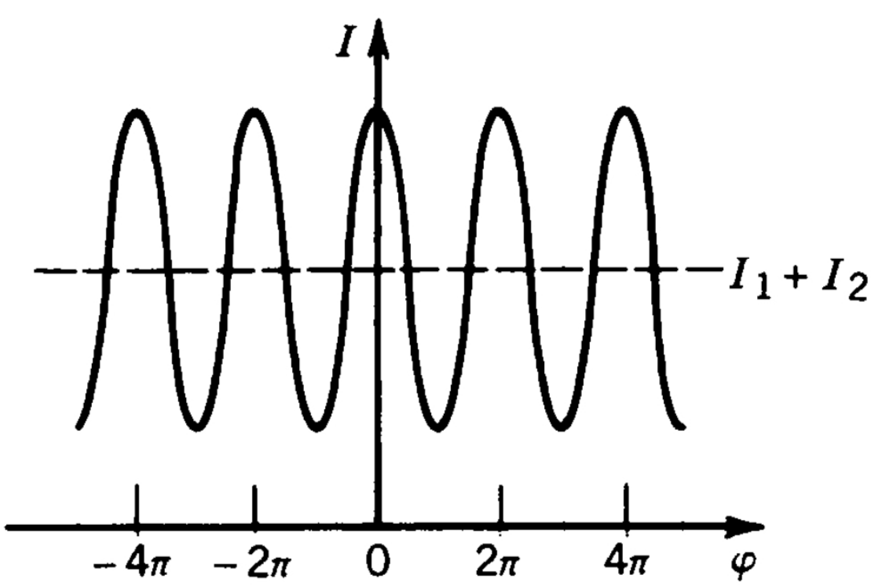
for $I_2 = I_1 = I_0$

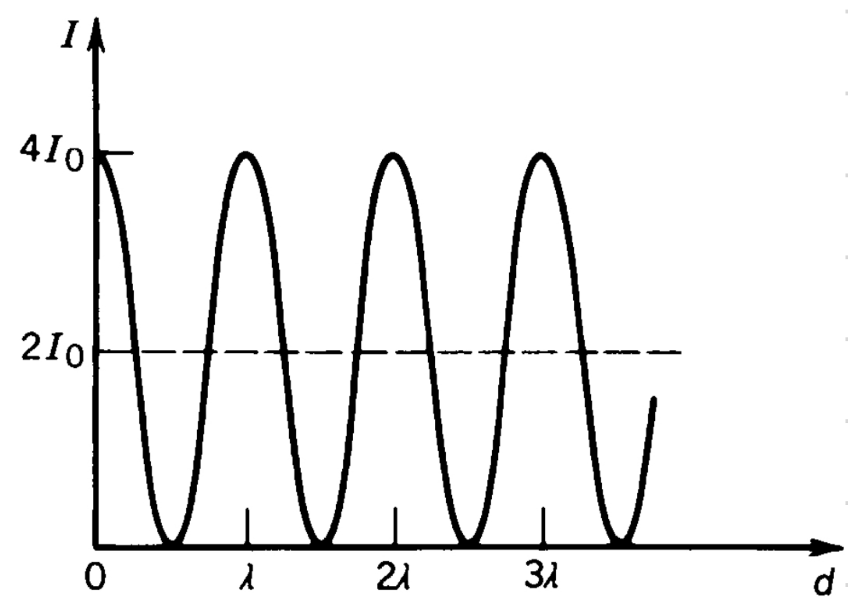
$$I = 2I_0(1 + \cos\varphi) = 4I_0\cos^2\left(\frac{\varphi}{2}\right)$$

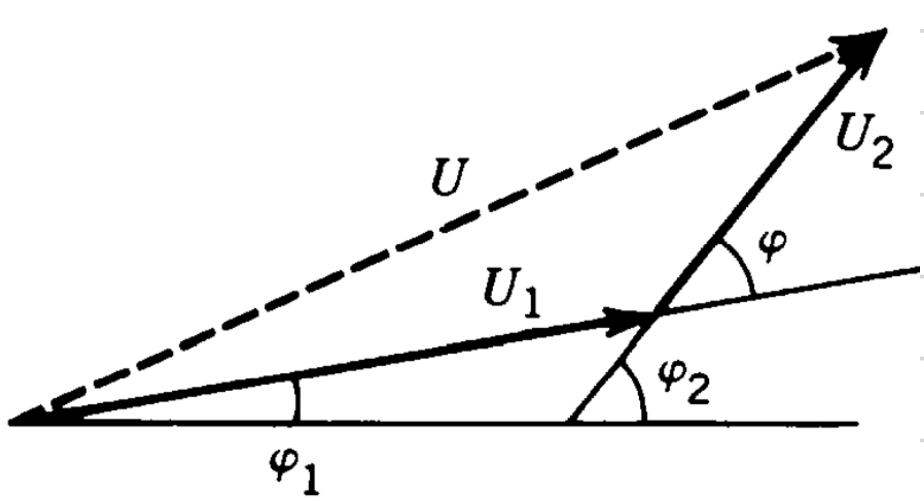
for $\varphi = 0 \rightarrow I = 4I_0$ constructive

$\varphi = \pi \rightarrow I = 0$ destructive

$\varphi = \pi/2\,,\,3\pi/2 \rightarrow I = 2I_0$







Phasor

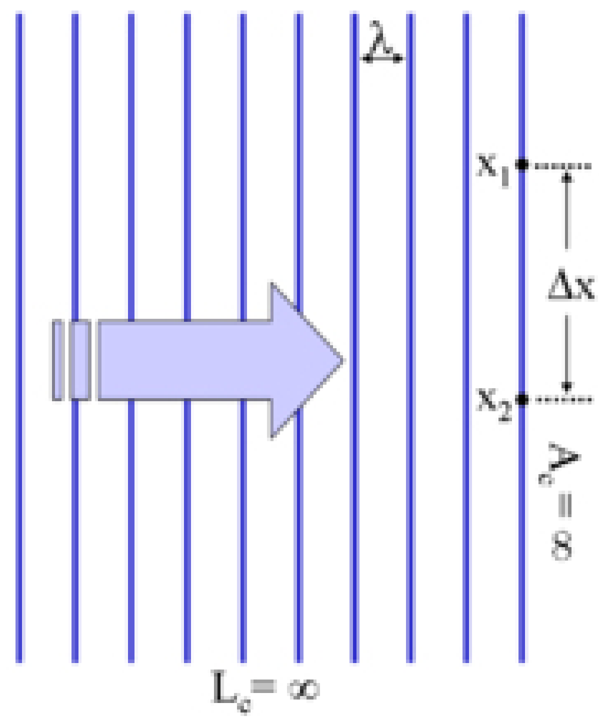
amplitude as

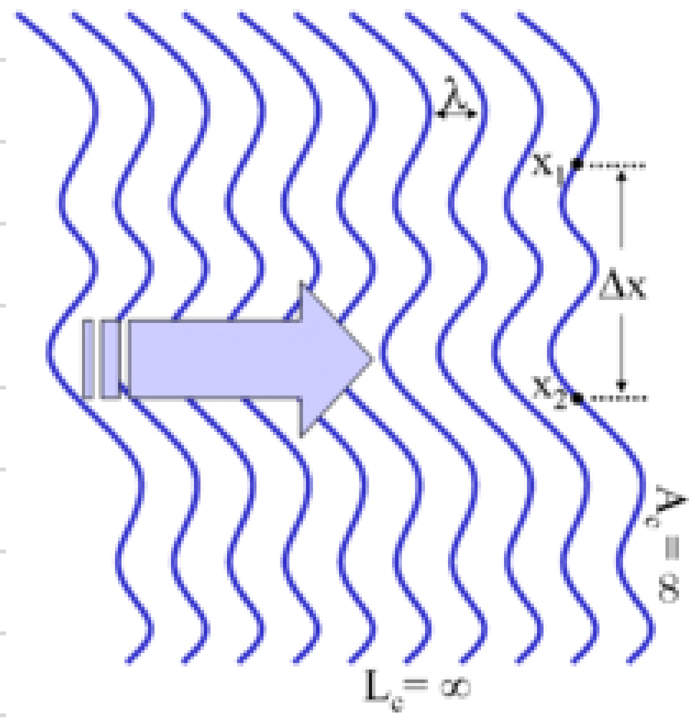
<u>Vector</u>

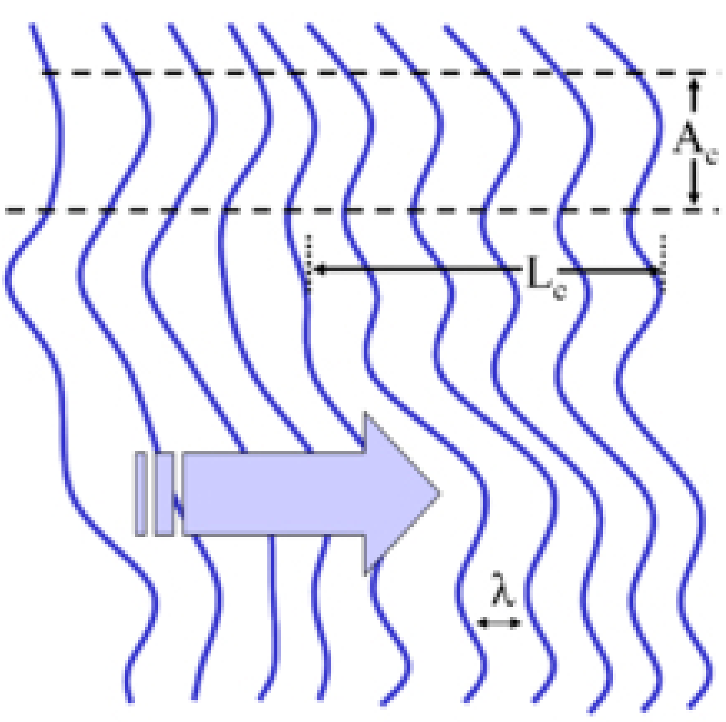
coherence: → Statistical Optics

spatial coherence:

Spatial coherence describes the ability for two points in space, x1 and x2, in the extent of a wave to interfere, when averaged over time.

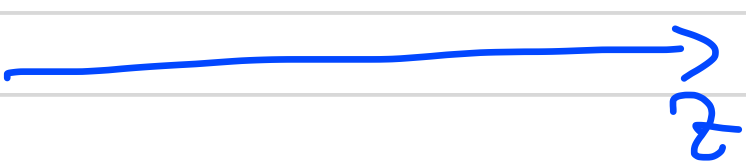


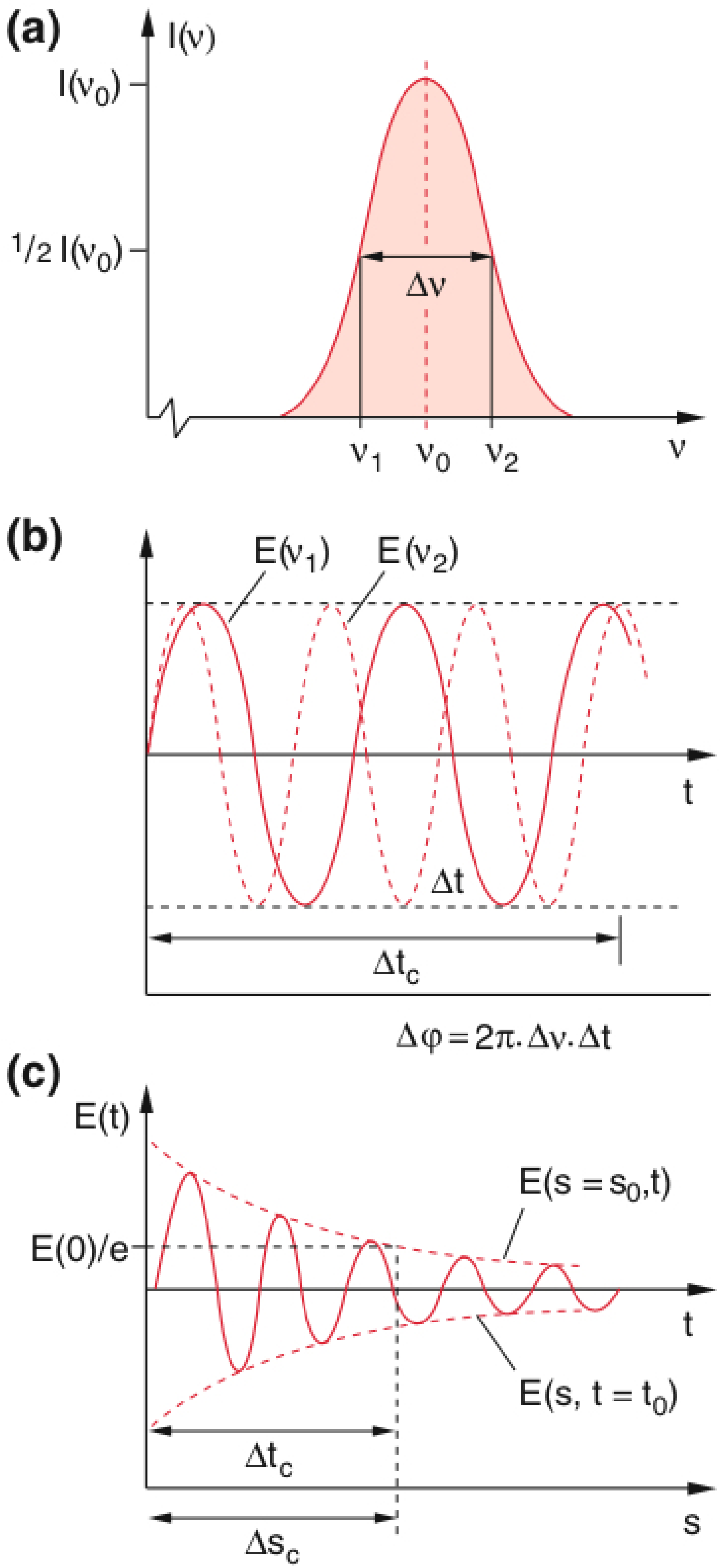




L_c is the coherence length

length over which the phase is not changing by more than 2π





$\Delta\varphi = 2\pi(\nu_2 - \nu_1)(t - t_0)$

$\nu_1 = \nu_0 - \Delta\nu/2$

$\nu_2 = \nu_0 + \Delta\nu/2$

after the coherence time $\Delta t_c = \frac{1}{\Delta\nu}$

$\Delta\varphi(\Delta t_c) > 2\pi$

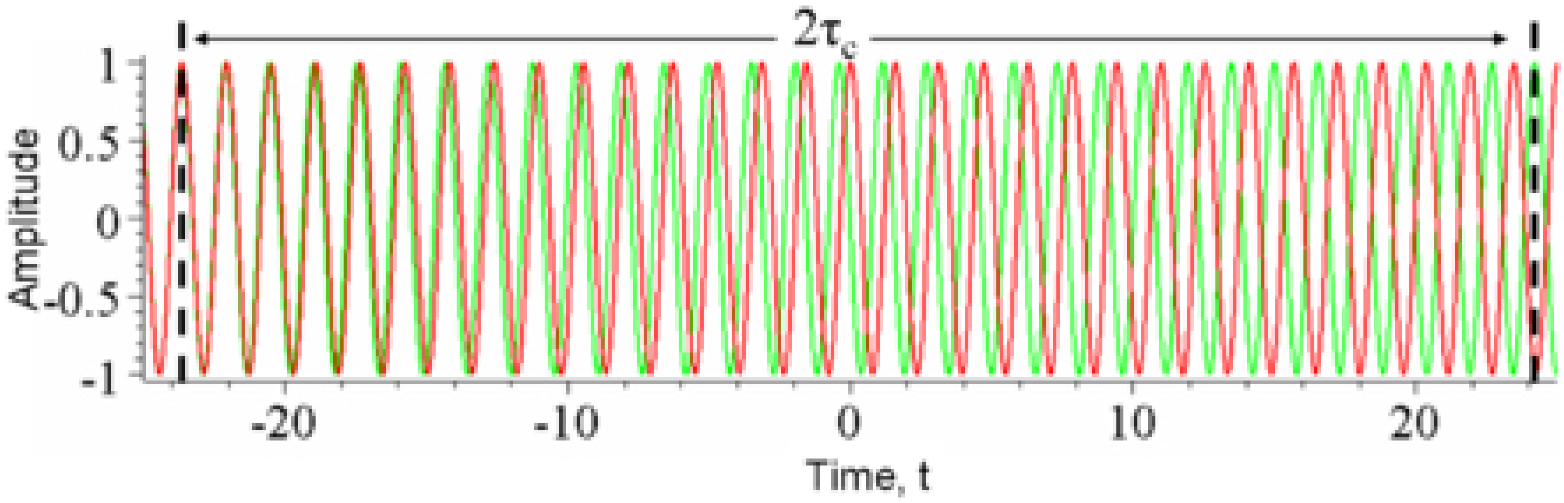
$\rightsquigarrow$ $\Delta t_c = \frac{1}{\Delta\nu}$ $\boxed{\text{Fourier relation}}$

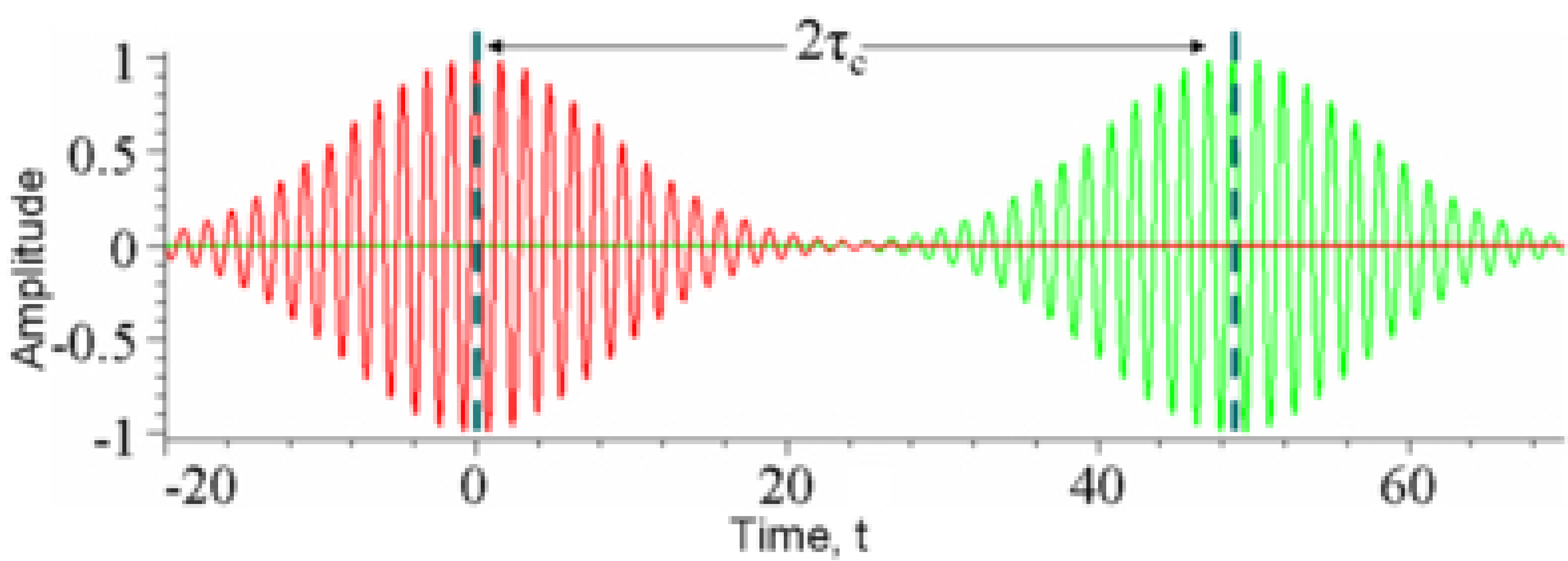
$\rightsquigarrow$ coherence length

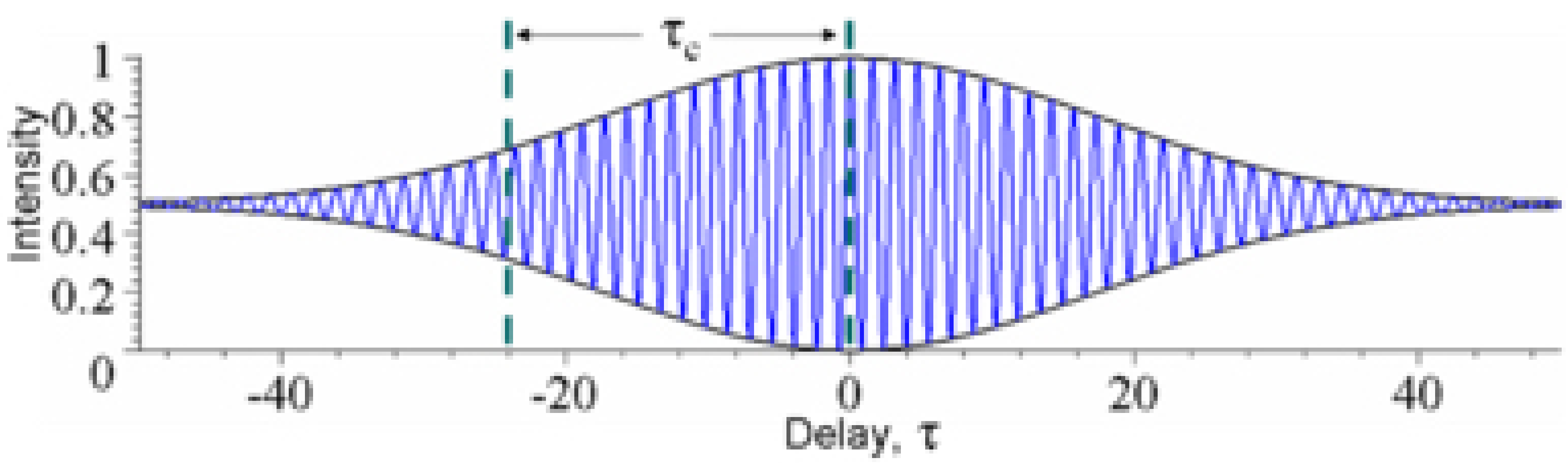
$L_c = c \cdot \Delta t_c$

Temporal Coherence

Temporal coherence is the measure of the average correlation between the value of a wave and itself delayed by τ, at any pair of times. Temporal coherence tells us how monochromatic a source is. In other words, it characterizes how well a wave can interfere with itself at a different time.

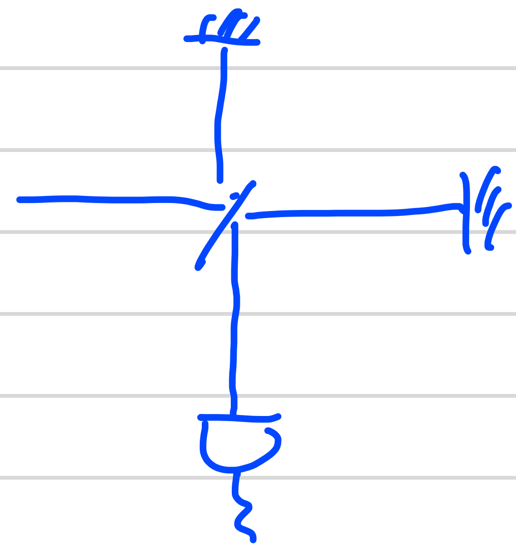




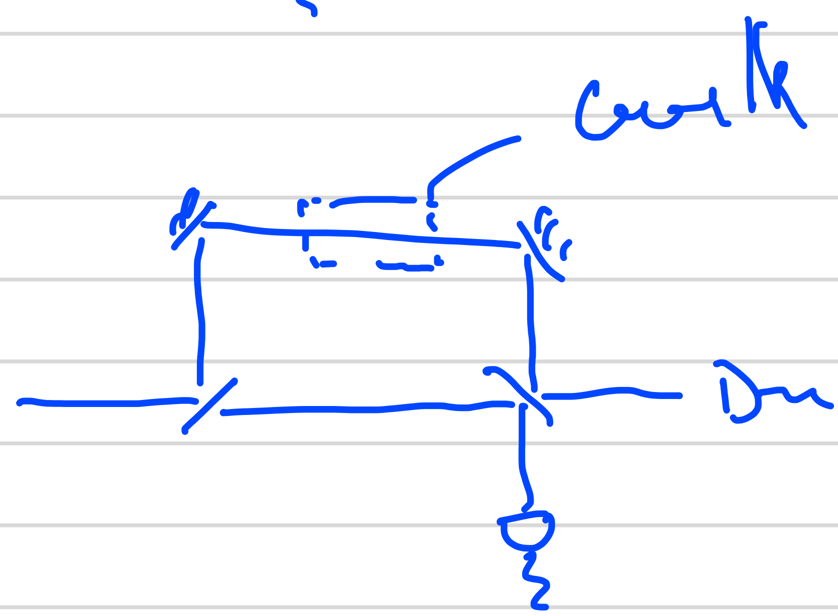


Interferometer:

Michelson



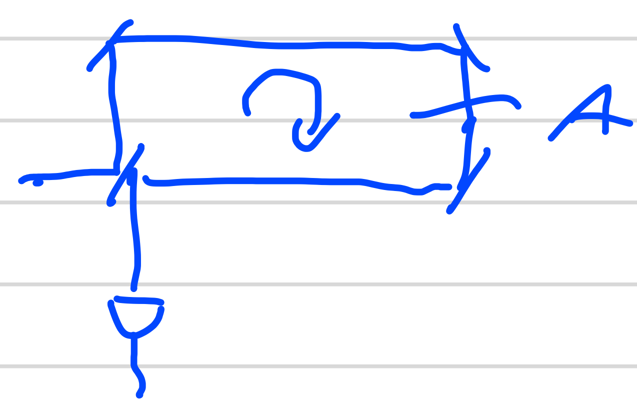
Mach Zehnder



$$\Delta S = \Delta S_0 + (n-1)L$$

empty cell

Sagnac



$dt = \frac{dl}{c}$, device turns by $\Omega \cdot dt$

shorter/longer by

$$dx = \Omega \cdot r\, dt$$

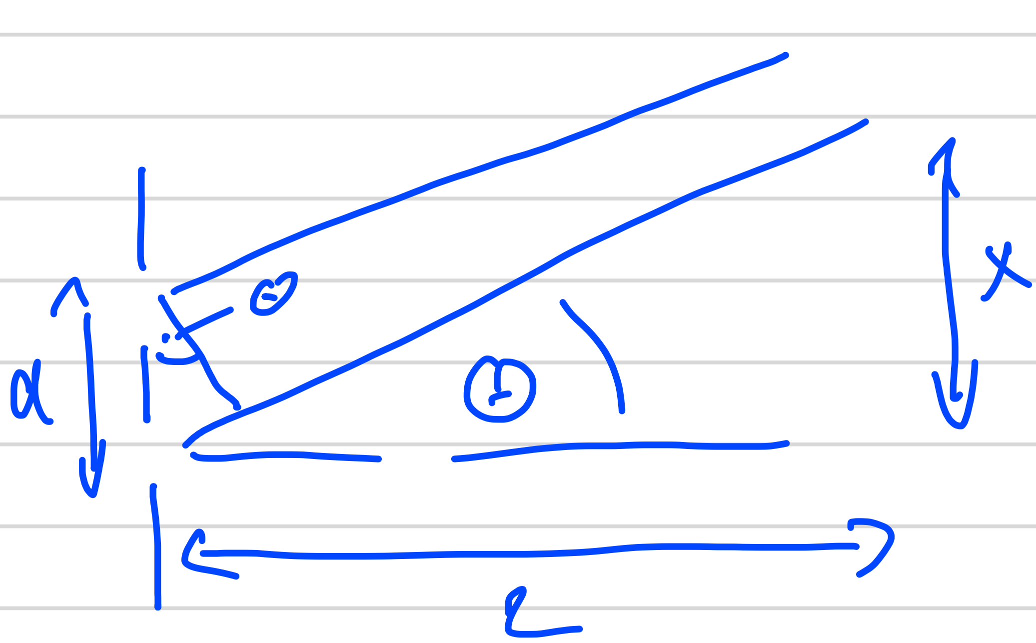
$$= \frac{\Omega \cdot r}{c}\, dl$$

$$x = \oint dx = \frac{\Omega}{c} \cdot 2A$$

$$\Delta\varphi = \frac{2\pi}{\lambda} \cdot 2x = \frac{8\pi A}{\lambda c}\Omega$$

$$\Delta\varphi = \frac{8\pi A}{c \cdot \lambda}\Omega \cdot \cos\Theta$$

Double Slit



$$\tan\Theta = \frac{\sin\Theta}{\cos\Theta} \approx \frac{x}{L}$$

$d \cdot \sin\Theta = m\lambda$ for constructive interference

$d \cdot \frac{x}{L} \approx m\lambda$

$d \cdot \sin\Theta = (m + \frac{1}{2})\lambda$ for destructive

$d \cdot \frac{x}{L} \approx (m + \frac{1}{2})\lambda$

discuss dependence on $\underline{d}$ and λ

$\rightarrow$ $U = U_1 + U_2$

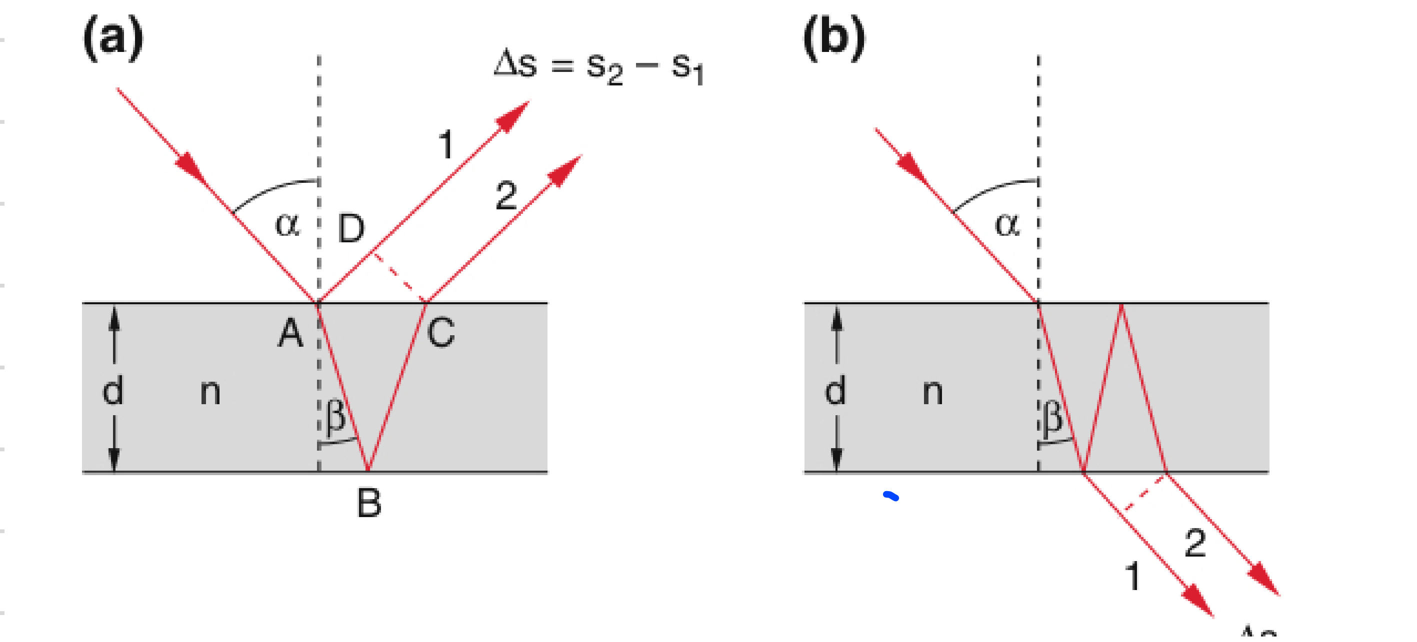
$$U = A \cdot e^{-ikr} + A e^{-ikr + i\varphi}$$

$$I = |U|^2$$

$$I = 4 I_0 \cdot \cos^2\left(\frac{\varphi}{2}\right)$$

$$= 4 I_0 \cos^2\left(\frac{\pi d \cdot \sin\Theta}{\lambda}\right)$$

Thin film interference



$$\Delta s = n(\overline{AB} + \overline{BC}) - \overline{AD}$$

$$= \frac{2nd}{\cos\beta} - 2d\tan\beta \cdot \sin\alpha$$

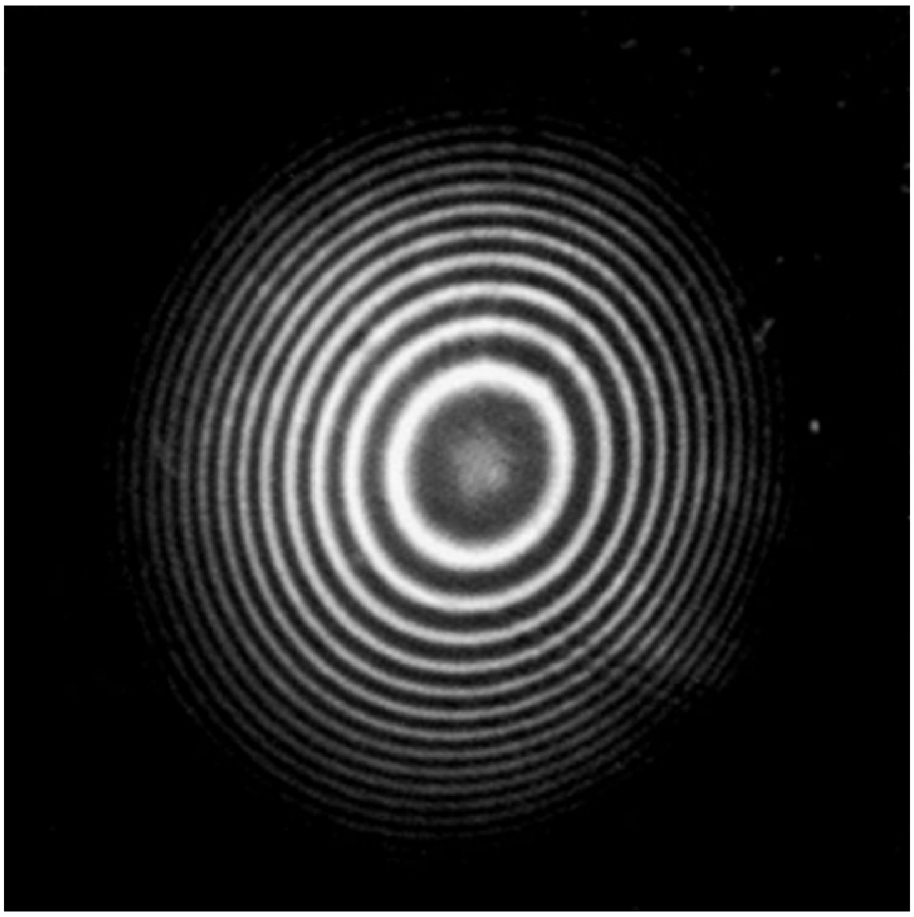
Snell $\sin\alpha = n \cdot \sin\beta$

$$\Delta s = \frac{2nd}{\cos\beta} - \frac{2nd\sin^2\beta}{\cos\beta} = 2nd\cos\beta$$

$$= 2d\sqrt{n^2 - \sin^2\alpha}$$

phase jump: $\Delta\varphi = \frac{2\pi}{\lambda}\Delta s + \pi$ reflection upper

normal



interference at parallel plate

$\Delta\varphi = m \cdot 2\pi$

$\rightsquigarrow$ constructive interference

1) $d \to 0$:

$\rightsquigarrow$ destructive interference $\boxed{\text{black}}$

2) constructive interference for , $\lambda = \text{const}$

$$d = \frac{(2m-1)\lambda}{4n}$$

3) find d but λ variable

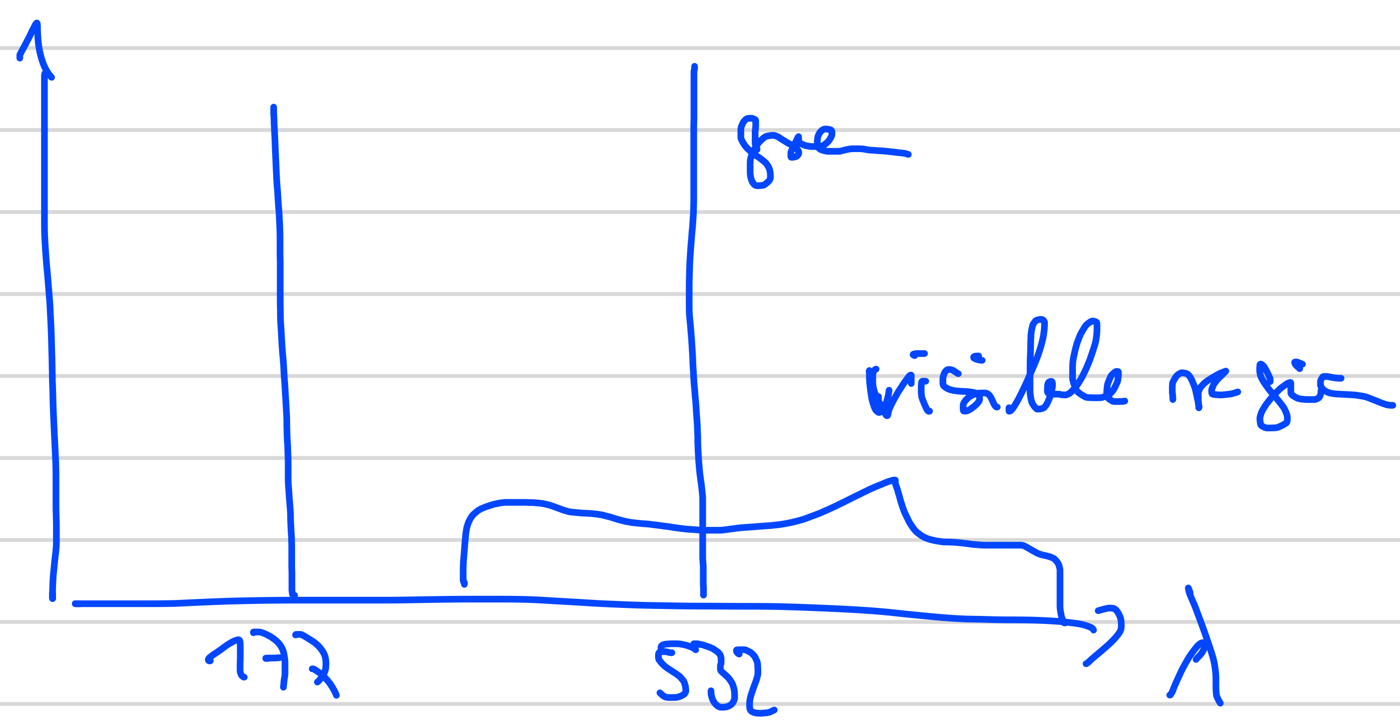
$$\lambda_{max} = \frac{4dn}{2m-1}$$

$d \sim 100\,nm$

$$\lambda_{max} = \frac{4 \cdot 100\,nm \cdot 1.33}{2m-1}$$

$$\lambda_{max} = \frac{532\,nm}{2m-1}$$

$\leadsto$ $m = 1$: $532\,nm$
$m = 2$: $177\,nm$
$m = 3$: $106\,nm$



$\leadsto$ $\lambda_{max} < 400\,nm$

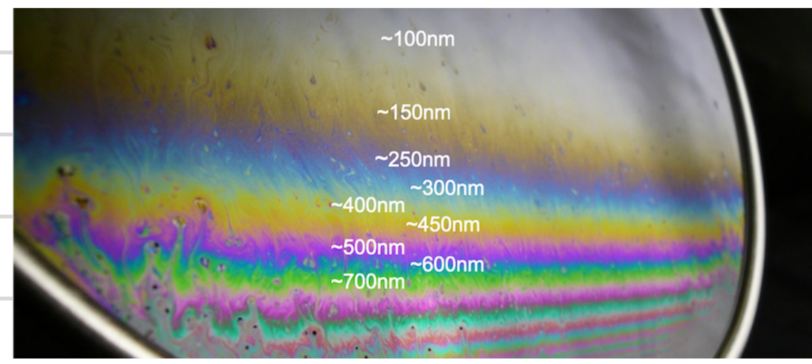
$$\leadsto d = \frac{(2m-1)\,\lambda_{max}}{4n} \approx 75\,nm$$

$\leadsto$ no reflection for $<75\,nm$ films ≙ Newton black film

$$d = 100\,\mu m$$

$$\lambda_{max} = \frac{532\,\mu m}{2m-1} \overset{?}{=} 532\,\mu m \quad \text{for } m=1$$

$\Updownarrow$ increase m



$$2m - 1 = \frac{532\,\mu m}{\lambda_{max}}$$

$$2m = \frac{532\,\mu m}{\lambda_{max}} + 1$$

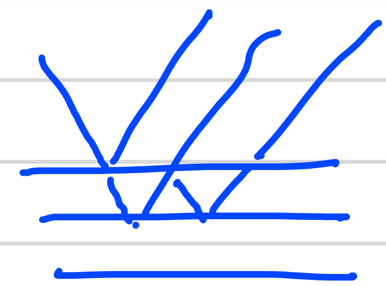
$$m = \frac{532\,\mu m}{2\lambda_{max}} + 1$$

$\lambda_{max} = 750\,nm \rightarrow m = 355$

$\lambda_{max} = 400\,nm \rightarrow m = 665$

white!

Multiple - Wave Interference (general)



$U = U_1 + U_2 + \dots U_M$

$I = |U|^2$

$$U_m = \sqrt{I_0}\, e^{i(m-1)\varphi} \qquad m = 1, 2, \dots, M$$

$$U = \sqrt{I_0}\,(1 + h + h^2 + \dots + h^{M-1})$$

$$= \sqrt{I_0}\, \frac{1-h^M}{1-h}, \qquad h = e^{i\varphi}$$

$$= \sqrt{I_0}\, \frac{1-e^{iM\varphi}}{1-e^{i\varphi}}$$

$$I = |U|^2 = I_0 \left| \frac{e^{-iM\varphi/2} - e^{iM\varphi/2}}{e^{-i\varphi/2} - e^{i\varphi/2}} \right|^2$$

$$= I_0 \frac{\sin^2(M\varphi/2)}{\sin^2(\varphi/2)}$$

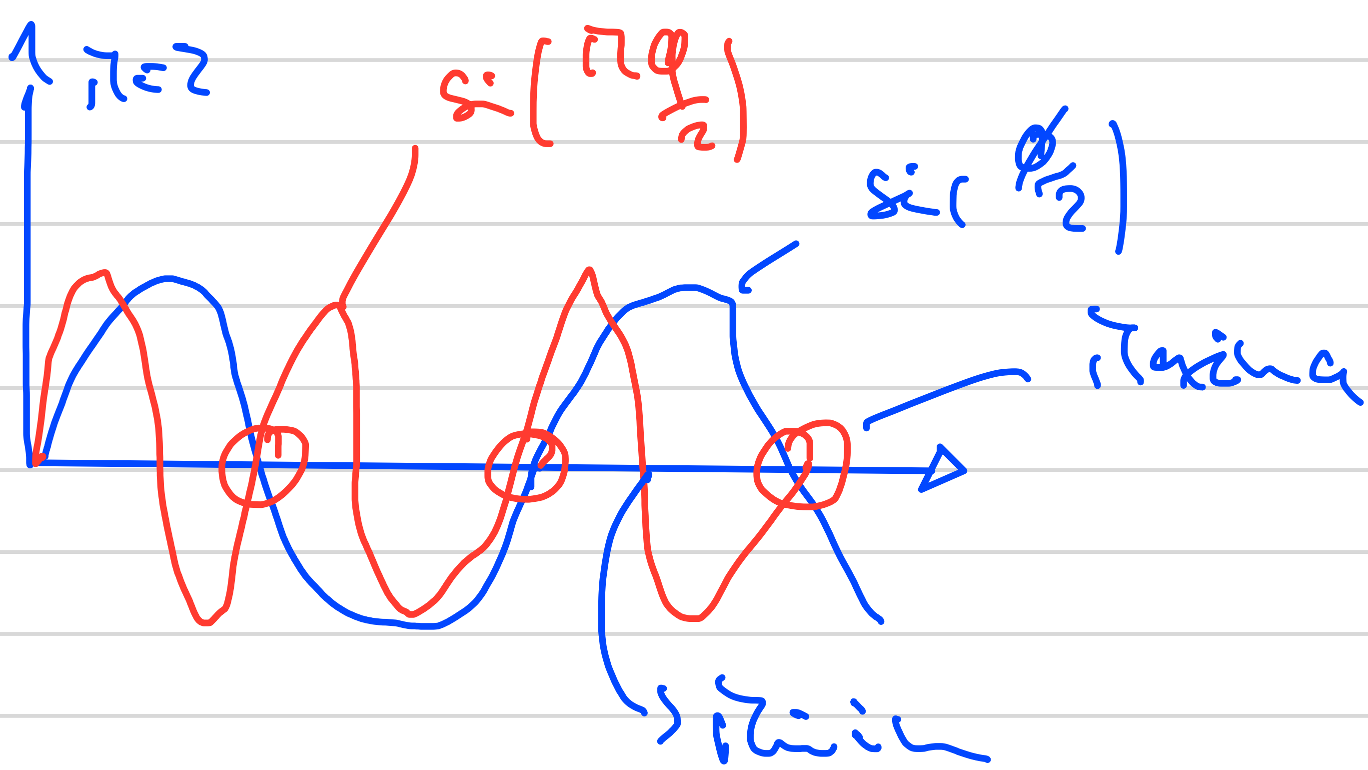
for $\varphi = 2\pi m \quad \rightarrow \quad M^2 I_0$

if $\varphi = 2\pi m$

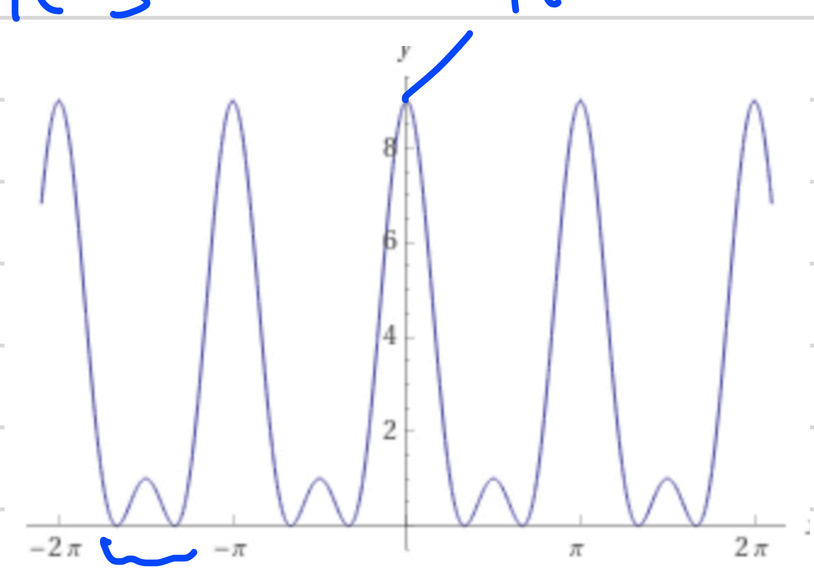
for $\varphi \to 0$

$$I = I_0 \cdot \left| \frac{N \, \varphi/2}{\varphi/2} \right|^2$$

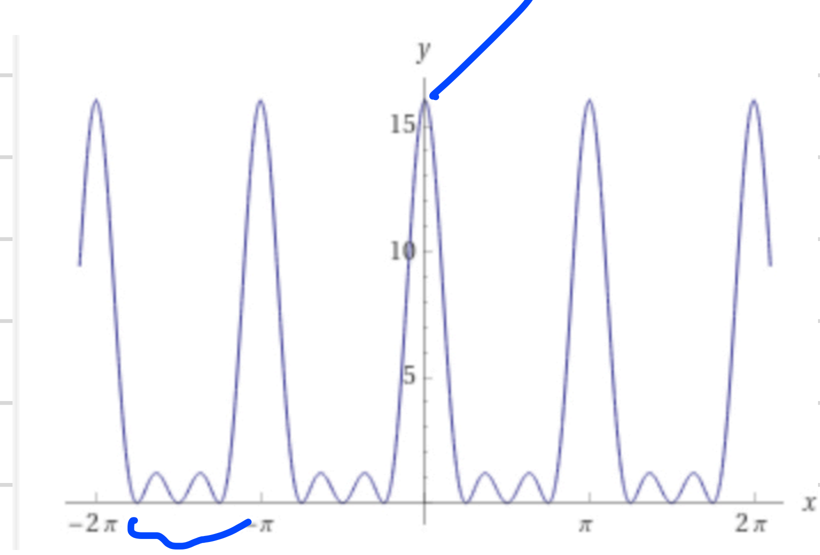
$$\underline{\underline{= N^2 I_0}}$$ → all intensity at maxima



N=3 N^2

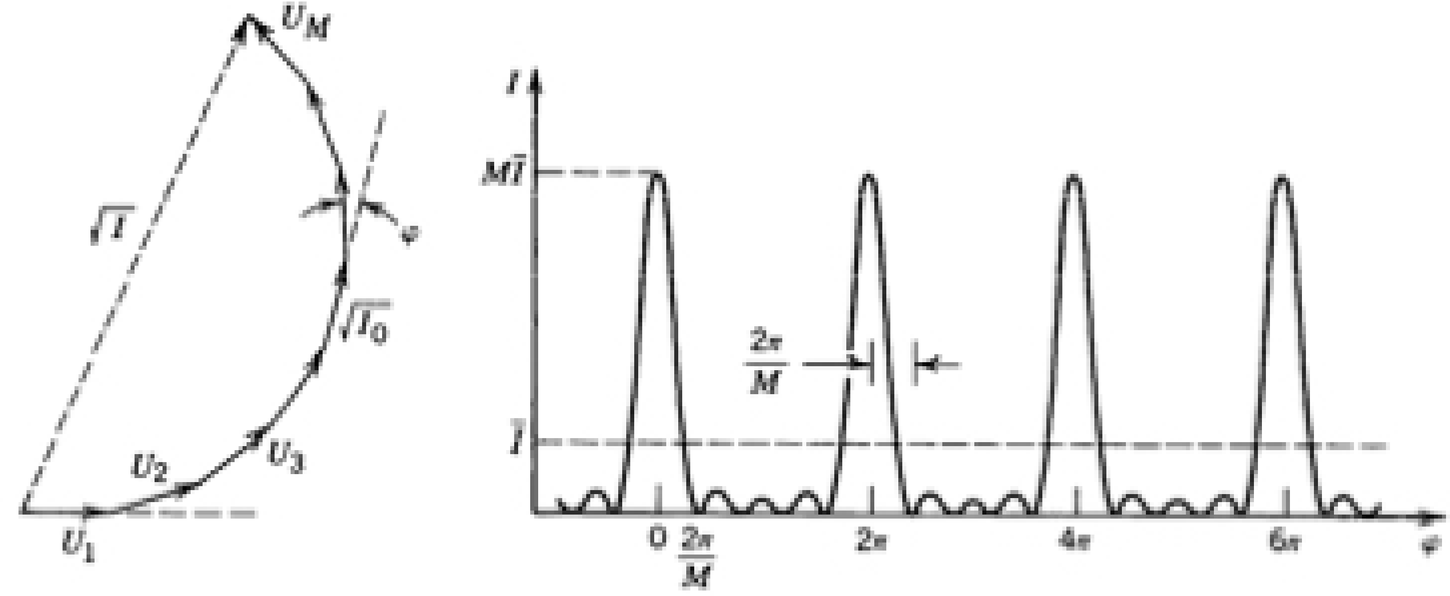


N=4 N^2

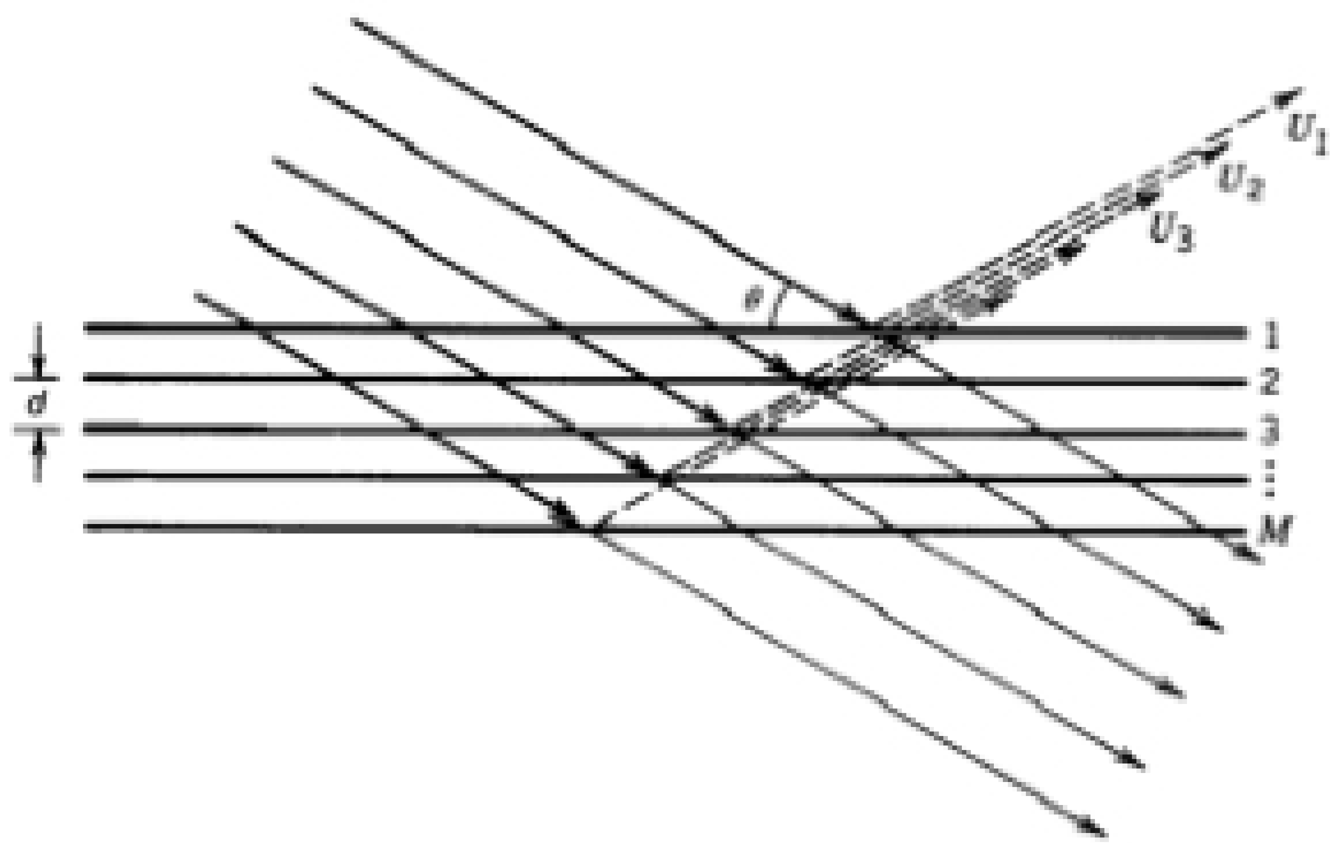


N-1 minima

N-1 minima



Bragg reflection (exp also with sound)



reflection has $\underline{\varphi = k(2d\sin\theta)}$ $\quad \sin\theta = \frac{m\cdot\lambda}{2d}$

$\varphi = 2\pi\cdot m$

this is now repeating all the same for slit, grating ...

Interference with decreasing amplitudes

$$U_1 = \sqrt{I_0} \qquad U_2 = h \cdot U_1 \qquad U_3 = h\, U_2 = h^2 U_1$$

$$h = r \cdot e^{i\varphi} \qquad |h| = r < 1 \quad \text{reflection}$$

$$U = U_1 + U_2 + U_3 + \dots$$

$$= \sqrt{I_0}\,(1 + h + h^2 + \dots)$$

$$= \frac{\sqrt{I_0}}{1-h} = \frac{\sqrt{I_0}}{1 - r e^{i\varphi}} = \frac{\sqrt{I_0}\,(1-h^{N})}{1-h}$$

$$I = |U|^2 = I_0 / |1 - r e^{i\varphi}|^2$$

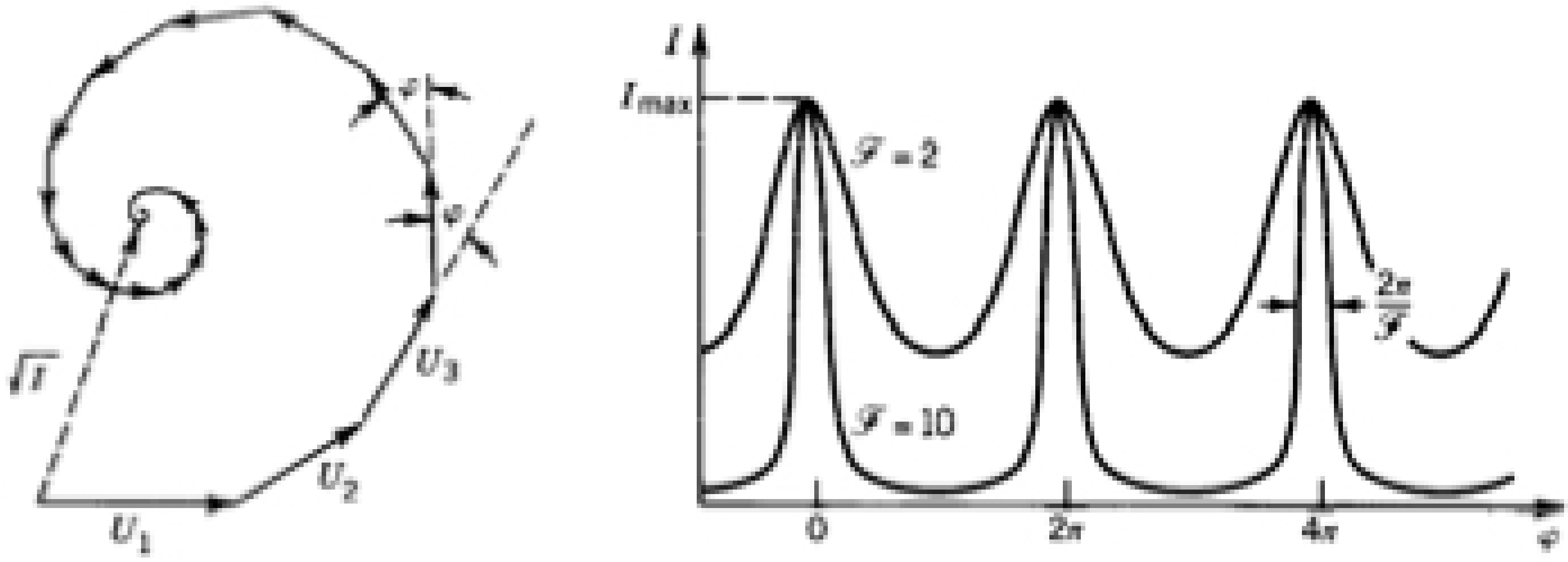
$$= \frac{I_0}{(1 - r\cos\varphi)^2 + r^2 \sin^2\varphi}$$

$$= \frac{I_0}{1 - 2r\cos\varphi + r^2\cos^2\varphi + r^2\sin^2\varphi}$$

$$= \frac{I_0}{1 - 2r\cos\varphi + r^2}$$

$$= \frac{I_0}{(1-r)^2 + 4r\sin^2(\varphi/2)} \qquad \text{Airy function}$$

$$I_{max} = \frac{I_0}{(1-r)^2}, \quad \mathcal{F} = \frac{\pi\sqrt{r}}{1-r} \Rightarrow \text{Finesse}$$



(a) (b)

$$I = \frac{I_{max}}{1 + 4\left(\frac{\mathcal{F}}{\pi}\right)^2 \sin^2\left(\frac{\varphi}{2}\right)}$$

at $\varphi = 0$: for $|\varphi| \ll 1$

$\sin(\varphi/2) \approx \varphi/2$

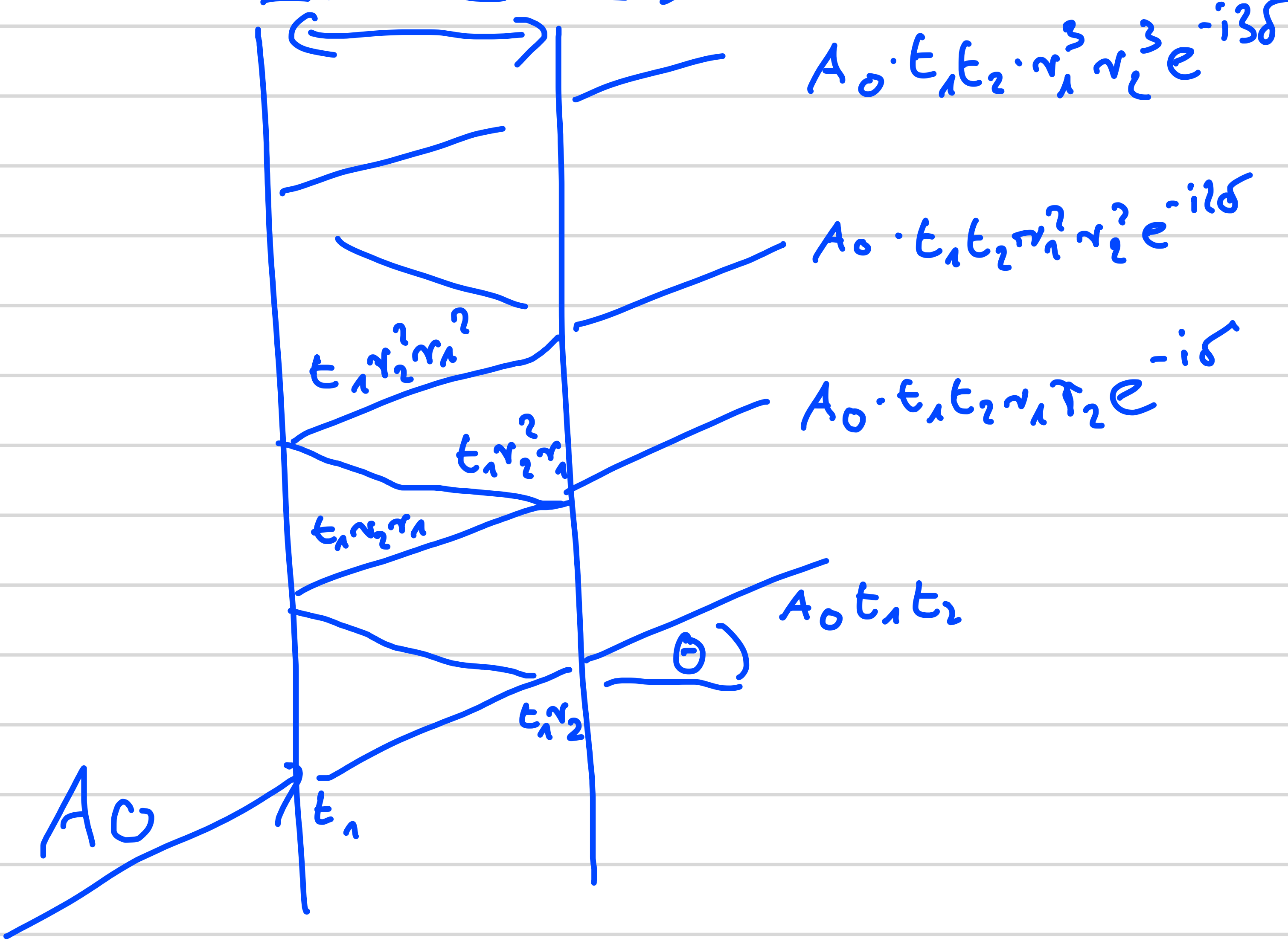
$$\rightsquigarrow I = \frac{I_{max}}{1 + \left(\mathcal{F}/\pi\right)^2 \varphi^2}$$

$$\rightsquigarrow \frac{I}{I_{max}} = 0.5 = \frac{1}{1 + \left(\frac{\mathcal{F}}{\pi}\right)^2 \varphi^2}$$

$$\boxed{\Delta\varphi = \frac{2\pi}{\mathcal{F}}}$$

Fabry Perot Interferometer

$$\Delta s = 2d \cdot \cos\theta$$

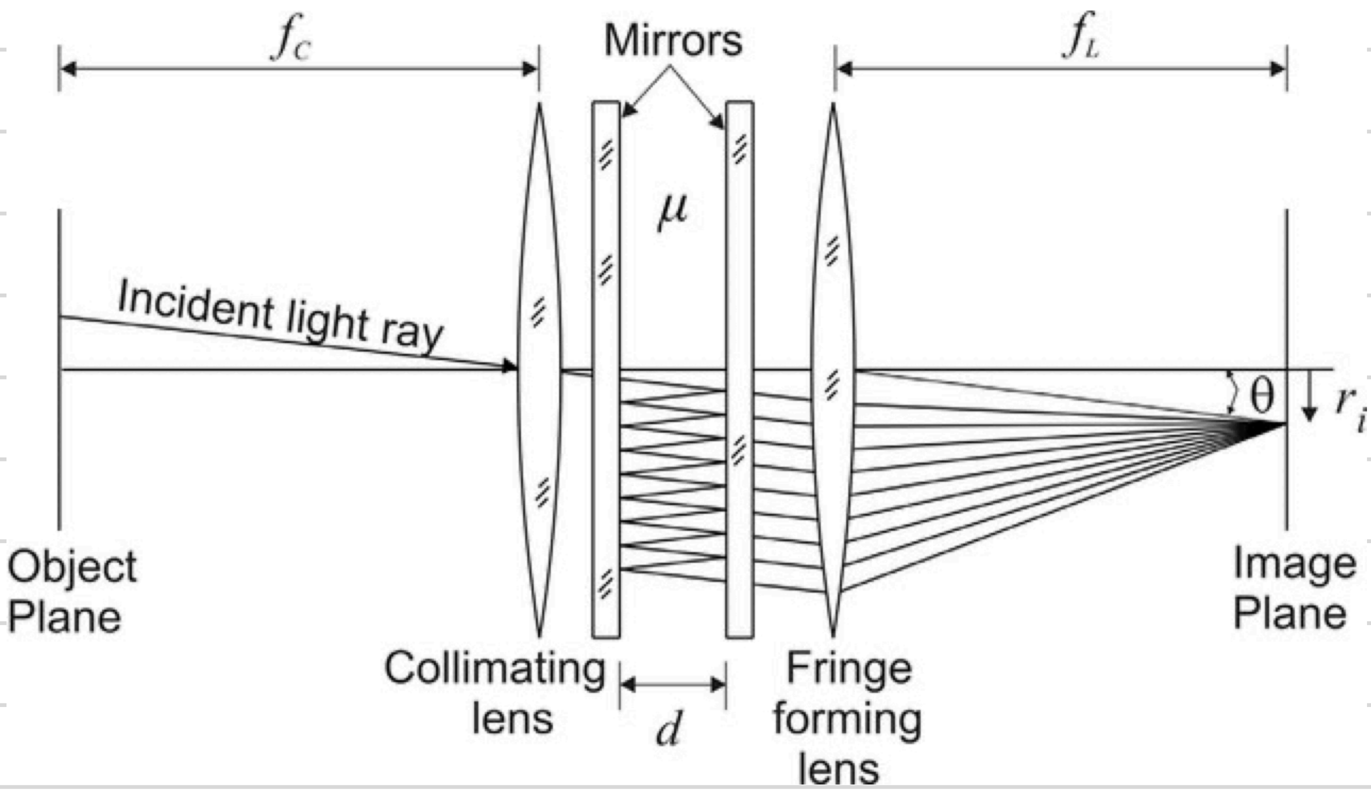


$$r_1 = r_2 = r: \quad A = A_0 t_1 t_2 \cdot \left[\frac{1}{1 - r e^{i\varphi}} \right]$$

$$I_t = \frac{I_0 T^2}{(1-r)^2} \left[\frac{1}{1 + 4\left(\frac{\tilde{F}}{\pi}\right)^2 \sin^2\left(\frac{\varphi}{2}\right)} \right]$$

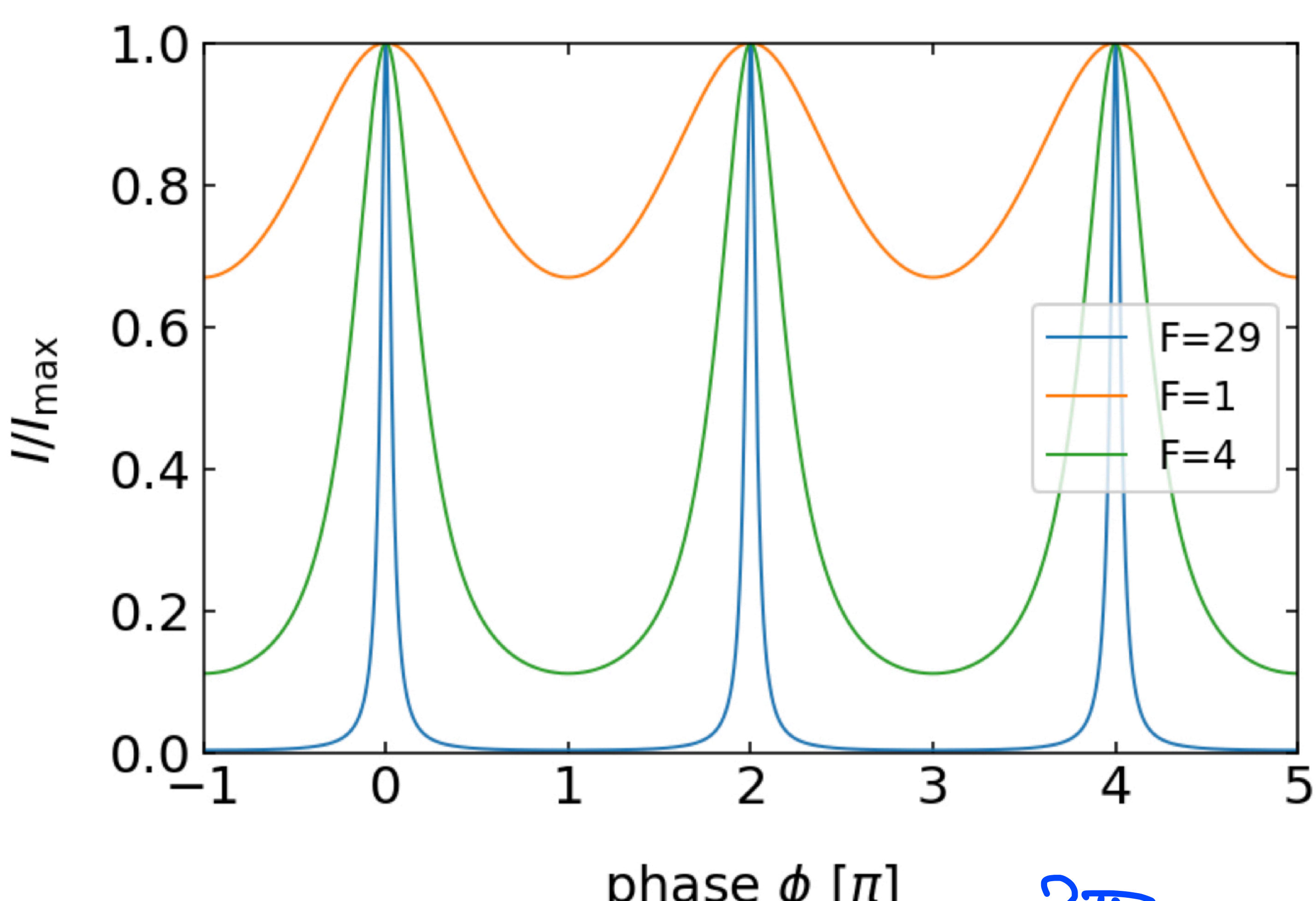
mit $\tilde{F} = \frac{\pi\sqrt{r}}{1-r}$

$$\Delta S = 2d\sqrt{n^2 - \sin^2\alpha} = 2d\cos\alpha = m\lambda$$

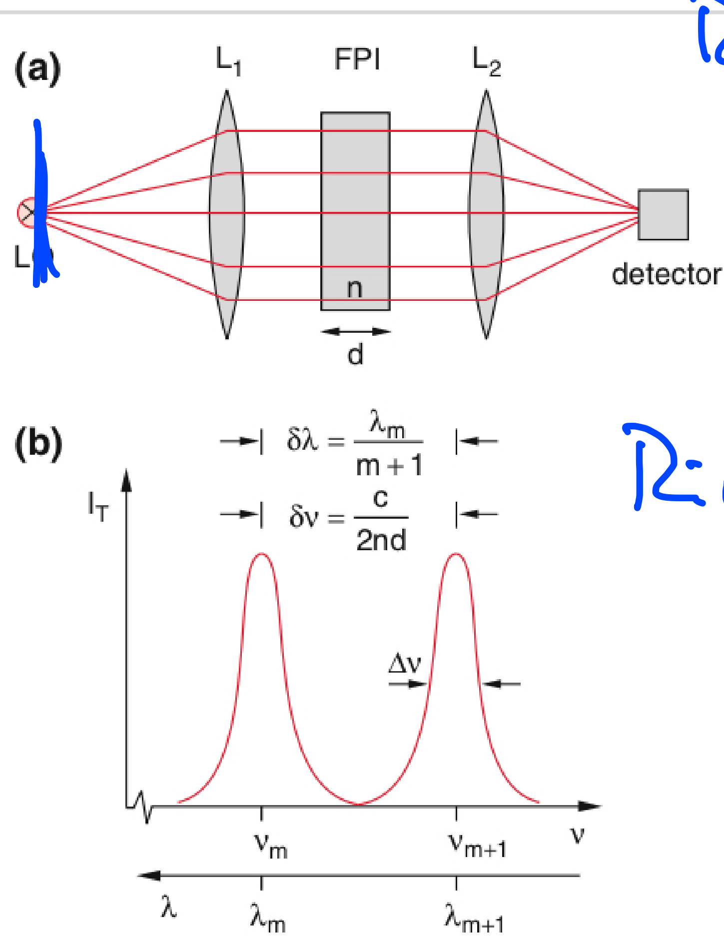


- fixing λ $\quad \Delta S = \frac{\lambda}{2\pi}\Delta\varphi$

- fixing ΔS $\quad \alpha$ varying α



$$\frac{2\pi}{\lambda}\cdot\Delta S = \varphi$$



Rings

Rings

$\underline{\alpha = 0}$ $\quad n = 1$

$$\Delta\varphi = \frac{2\pi}{\lambda}\cdot \Delta s$$

$$= \frac{4\pi}{\lambda}\cdot d = m 2\pi$$

$$\Delta\varphi = m\cdot 2\pi \quad \underline{\underline{\text{const.}}}$$

$$\rightsquigarrow \lambda_m = \frac{2d}{m}$$

$$\nu = \frac{c}{\lambda}, \quad \varphi = \frac{4\pi\nu}{c} d$$

free spectral range

$$\nu_{m+1} = \frac{c(m+1)}{2d}, \quad \nu_m = \frac{cm}{2d}$$

$$\boxed{\delta\nu = \nu_{m+1} - \nu_m = \frac{c}{2d}}$$

$$\delta\lambda = \lambda_m - \lambda_{m+1} = \frac{2d}{m} - \frac{2d}{m+1}$$

$$= \frac{2d}{m(m+1)} = \frac{\lambda_m}{m+1}$$

$\delta\nu, \delta\lambda$ ... $\underline{\text{free spectral range}}$

half width of the peak

$$I(\nu_1) = I(\nu_2) = 0.5\, I(\nu_m)$$

$$I = \frac{I_{max}}{1 + \left(\tilde{F}/\pi\right)^2 \Delta\varphi^2}$$

$$\frac{I}{I_{max}} = 0.5 = \frac{1}{1 + \frac{F^2}{\pi^2}\Delta\varphi_{1/2}^2}$$

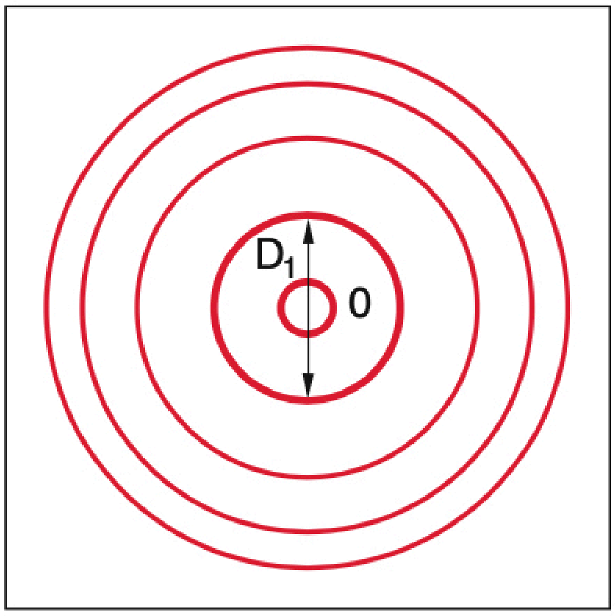
$$1 + \left(\frac{\tilde{F}}{\pi}\right)^2 \Delta\varphi_{1/2}^2 = 2$$

$$\Delta\varphi_{1/2}^2 = \frac{2\pi^2}{\tilde{F}^2}$$ ⤳ Full width

$$\rightsquigarrow \frac{4\pi\,\nu_{1/2}\,d}{c} = \frac{2\pi}{\tilde{F}} \Rightarrow \underline{\nu_{1/2} = \frac{c}{2d\tilde{F}} = \frac{\delta\nu}{\tilde{F}}}$$

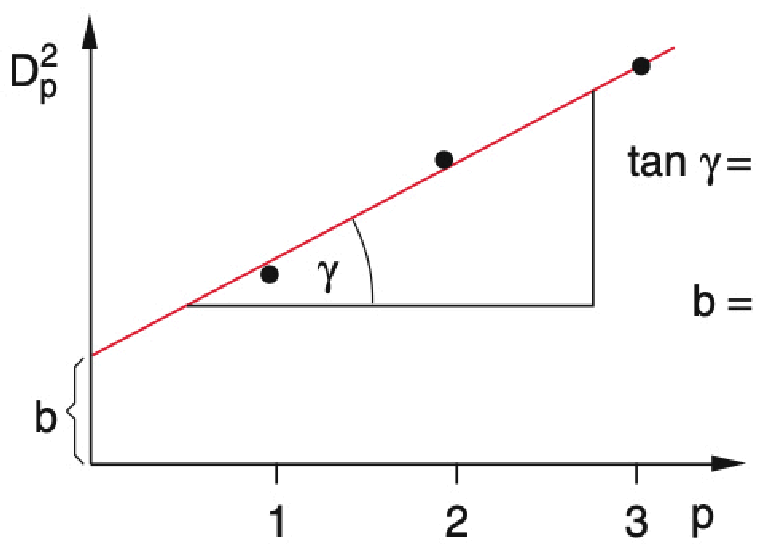
$$\boxed{\tilde{F} = \frac{\delta\nu}{\Delta\nu} = \frac{\lambda}{d\lambda}}$$ Finesse is related to the resolving power

$$\boxed{R = m\tilde{F} \quad \text{resolving power}}$$



$$D_p^2 = \frac{4f_2^2 \cdot \lambda}{d}(p + \varepsilon)$$

$$D_p = 2 f_2 \cdot \tan \alpha_p$$
$$\approx 2 f_2 \cdot \alpha_p$$

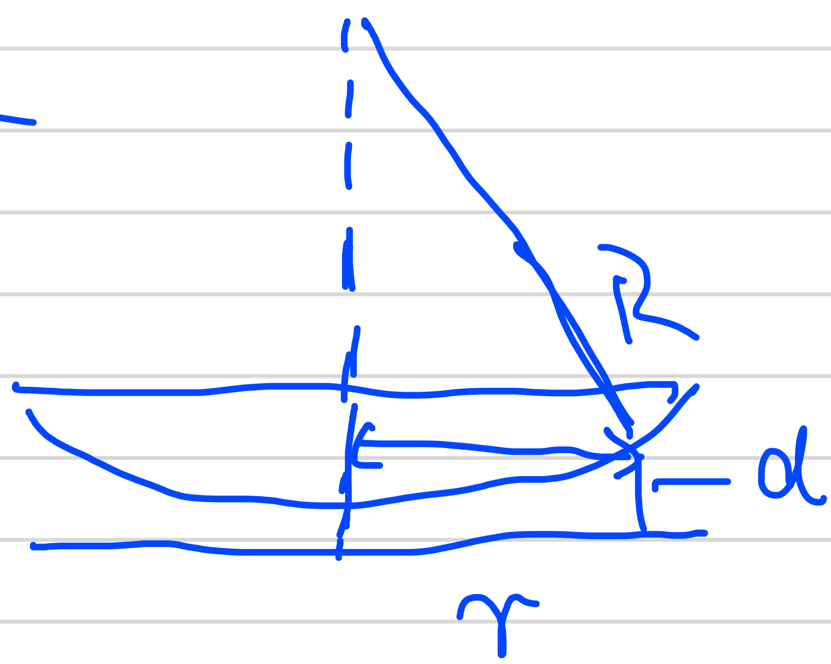


$$\tan \gamma = \frac{4f_2^2 \cdot \lambda}{d}$$

$$b = \frac{4f_2^2 \cdot \lambda}{d} \cdot \varepsilon$$

slope allows the det. of λ

Newton Rings



$$\Delta s = 2d + \frac{\lambda}{2}$$

destructive interference for

path difference

$$\Delta S = \frac{2m+1}{2} \cdot \lambda = 2d + \frac{\lambda}{2}$$

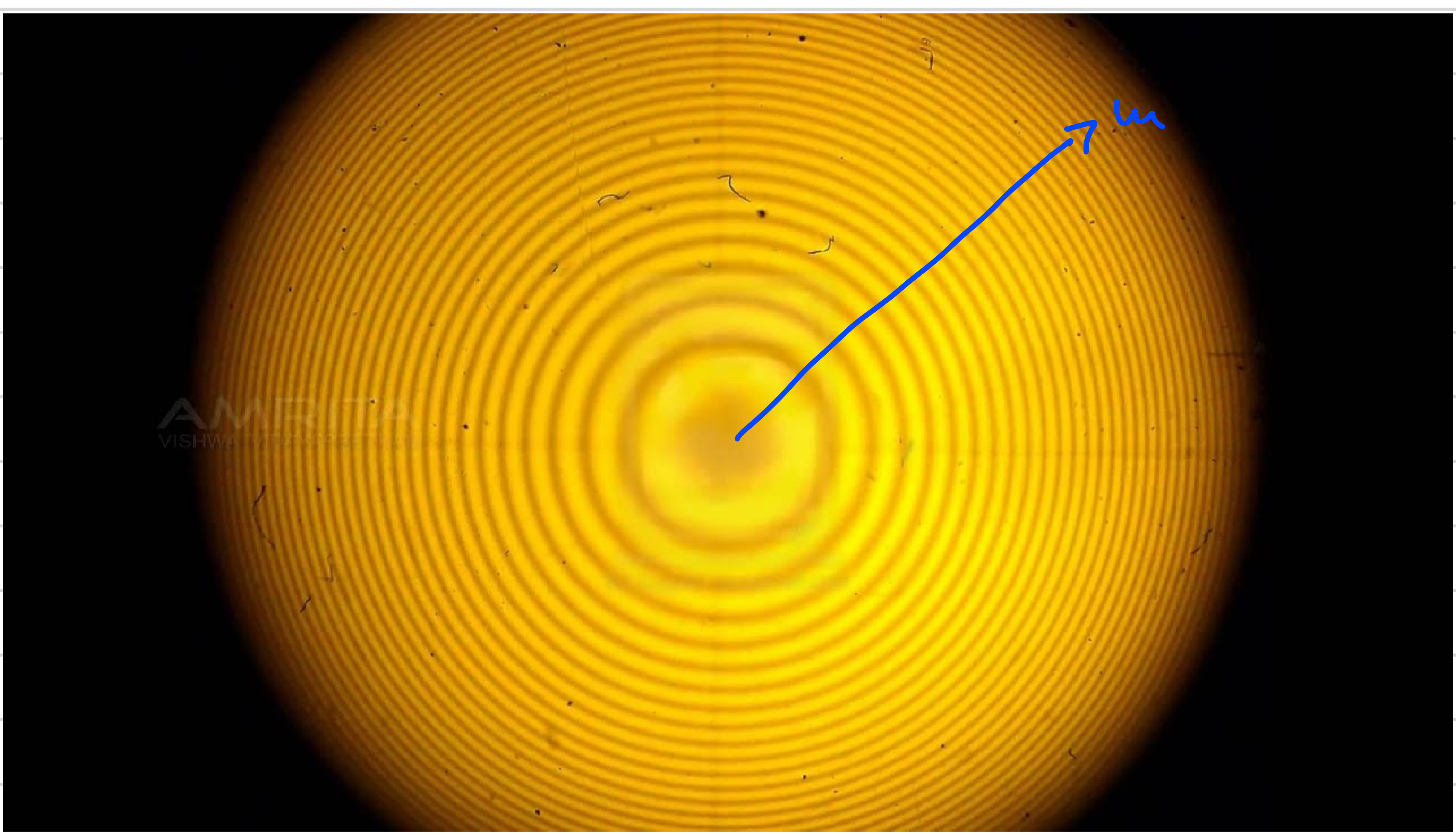
$$\leadsto 2d = m \cdot \lambda$$

radius of disk. $r^2 = d(2R - d)$

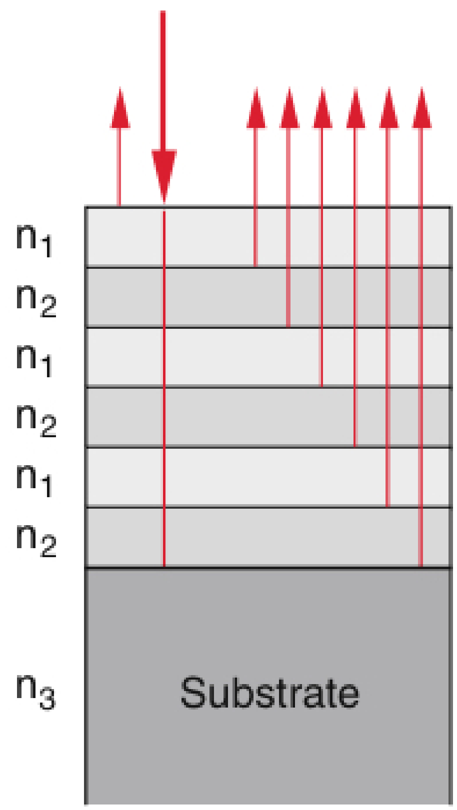
$r^2 = 2dR$ for $d \ll R$

$$\leadsto d = \frac{r^2}{2R}$$

$$\leadsto 2\frac{r^2}{2R} = m\lambda \quad \Rightarrow \quad r = \sqrt{m\lambda R}$$



Dielectric Mirrors (constructive interference)

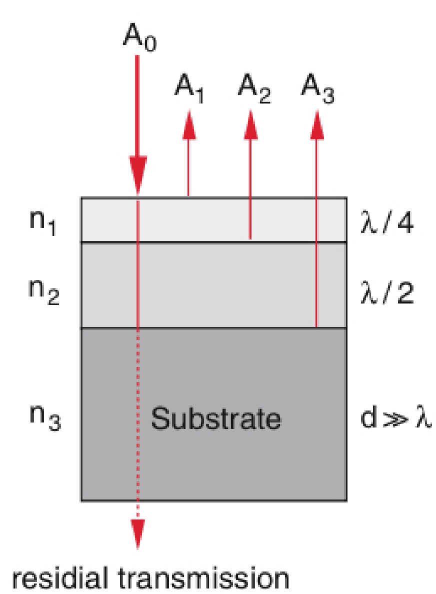


generate a stack of thin films with spec. refractive index to design specific reflectivity

for example

$n_{air} < n_1 > n_2, n_3$

constructive interference:



$$n_1 \cdot d_1 = \frac{\lambda}{4}$$

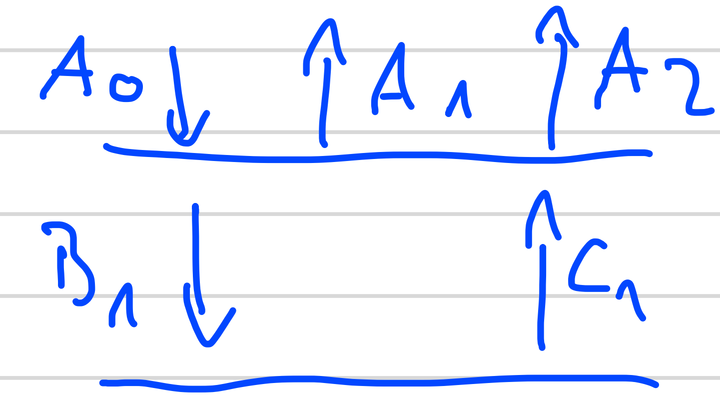
$$n_2 \cdot d_2 = \lambda/2$$

reflection coefficient

$$R_1 = \left(\frac{n_1 - 1}{n_1 + 1}\right)^2$$

$$R_2 = \left(\frac{n_1 - n_2}{n_1 + n_2}\right)^2 , \quad R_3 = \left(\frac{n_2 - n_3}{n_2 + n_3}\right)^2$$

individual reflections



$$\sqrt{R_1} = r_1$$

$$|A_1| = \sqrt{R_1} \cdot |A_0|$$

$$|B_1| = (1-\sqrt{R_1})|A_0|$$

$$|C_1| = (1-\sqrt{R_1})\sqrt{R_2}\, A_0$$

$$|A_2| = (1-R_1)\sqrt{R_2}\,|A_0|$$

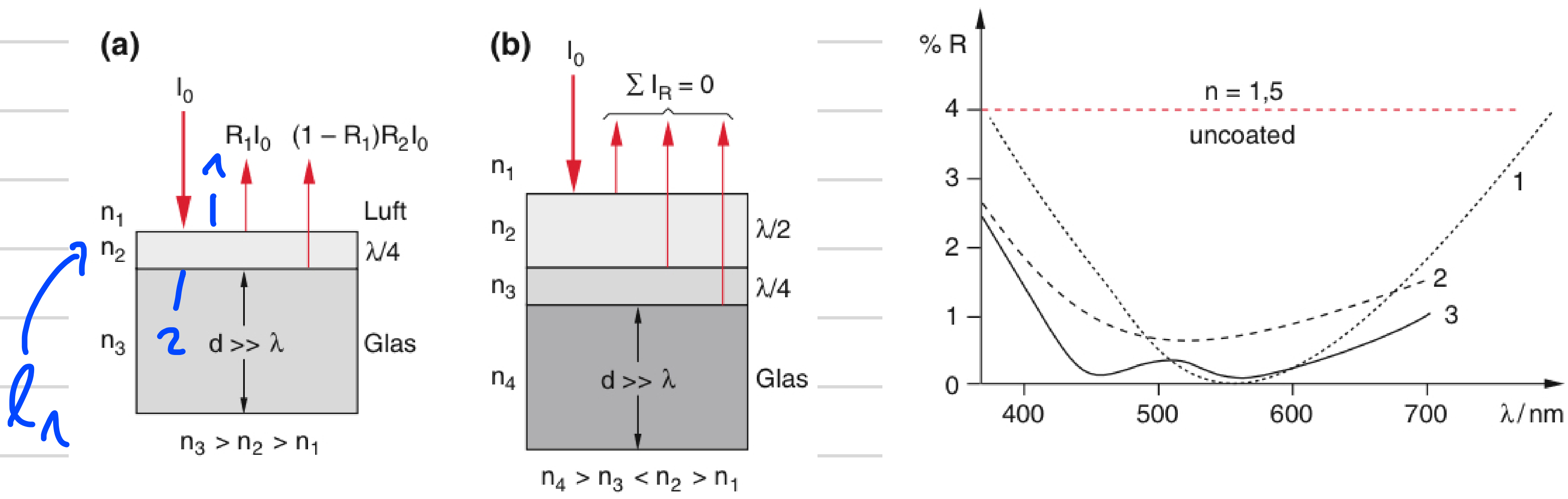
$$R_1 I_0 = (I_0 - \underbrace{R_1 I_0}_{\text{small}}) R_2$$

$$R_1 I_0 = I_0 R_2 - \underbrace{R_1 R_2 I_0}_{\text{small}}$$

$$R_1 = R_2$$

$$\frac{n_2-1}{n_2+1} = \frac{n_3-n_2}{n_3+n_2}$$

Anti-Reflex Coating! $\Delta\varphi = (2m+1)\pi$



single Layer

$n_1 < n_2 < n_3$

⇝ phase flip on 1, 2

$2k \cdot l_1 = (2m+1)\pi$

for destructive interference

$$\frac{4\pi}{\lambda} l_1 = (2m+1)\pi \Rightarrow l_1 = (2m+1)\frac{\lambda}{4}$$

$m = 0, 1 ..$

so first at $l_1 = \frac{\lambda}{4}$

then $R_1 I_0 = (1-R_1) R_2 I_0$

$$R_1 I_0 \approx R_2 \cdot I_0$$

$$\leadsto \quad R_1 = R_2$$

$$\frac{n_2 - n_1}{n_2 + n_1} = \frac{n_3 - n_2}{n_3 + n_2}$$

$$(n_2 - n_1)(n_3 + n_2) = (n_3 - n_2)(n_2 + n_1)$$

$$\cancel{n_2 n_3} + n_2^2 - n_1 n_3 - \cancel{n_1 n_2}$$

$$= \cancel{n_3 n_2} + n_3 n_1 - n_2^2 - \cancel{n_2 n_1}$$

$$\cancel{2} n_2^2 = \cancel{2} n_1 n_3$$

$$n_2 = \sqrt{n_1 n_3}$$

for $n_1 = 1$, $n_3 = 1.5$ $\leadsto$ $n_2 = \sqrt{1.5} = 1.22$

Na_3AlF_6 $\quad n_1 = 1.35$

MgF_2 $\quad n_1 = 1.38$ $\quad$ best dielectric

<u>but narrow wavelength range</u>

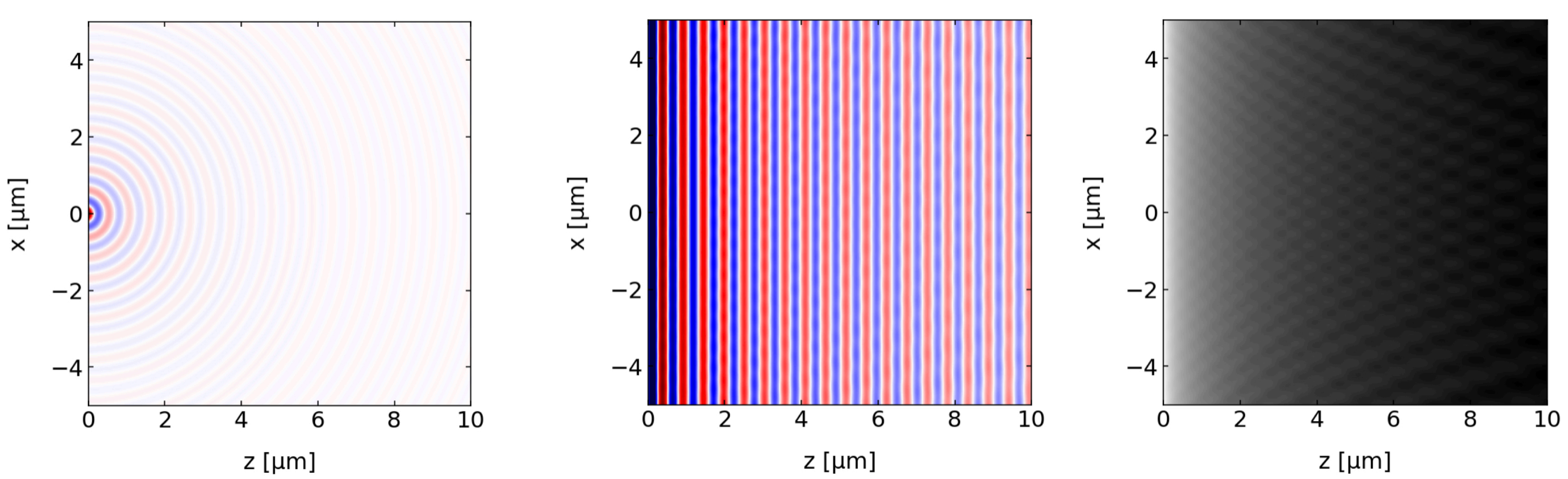
2.3 Diffraction

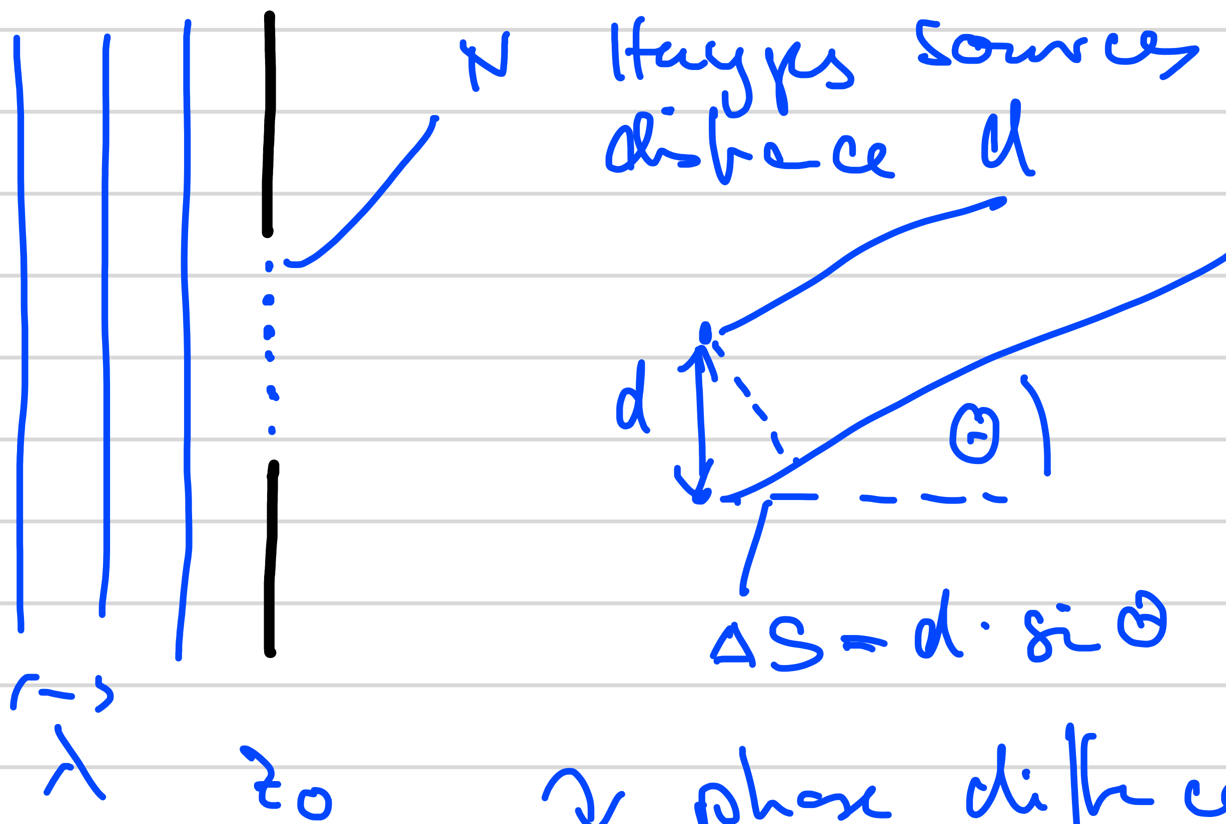
2.3.1 Huygens Principle

- Every point on a wavefront may be considered a source of secondary spherical wavelets, which spread out in the forward direction at the speed of light

  → the new wavefront is tangential to all wavefronts

reconstruction of a plane wave





↝ phase difference

$$\Delta\varphi = \frac{2\pi}{\lambda}\cdot\Delta S = \frac{2\pi}{\lambda}\, d\cdot\sin\Theta$$

$$I(\Theta) = I_0\, \frac{\sin^2\left(N\frac{\Delta\varphi}{2}\right)}{\sin^2\left(\frac{\Delta\varphi}{2}\right)}$$

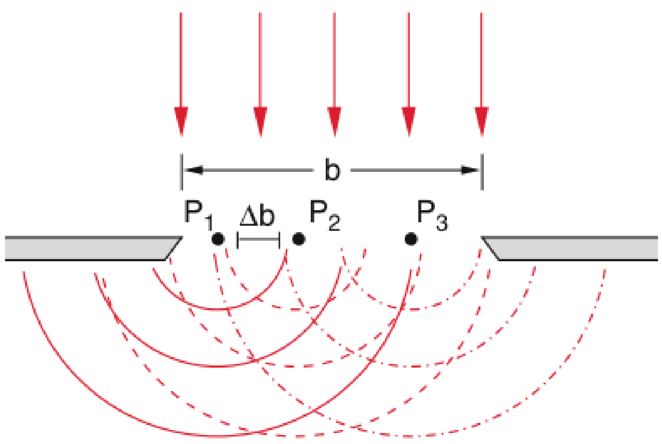
<u>interference from N sources</u>

$$\boxed{I = I_0 \cdot \frac{\sin^2\left(N\frac{\pi}{\lambda}\cdot d \sin\Theta\right)}{\sin^2\left(\frac{\pi}{\lambda}\cdot d \sin\Theta\right)}}$$

→ numerator

→ denominator

2.3.2 Single and multiple slit diffraction



single slit

line segment Δb

$\rightsquigarrow$ $N = \frac{b}{\Delta b}$ oscillators

each emitter has A_0 amplitude

$$\rightsquigarrow \quad I(\Theta) = I_0 \frac{\sin^2\left(\pi \cdot \frac{b}{\lambda} \cdot \sin\Theta\right)}{\sin^2\left(\pi \frac{\Delta b}{\lambda} \sin\Theta\right)}$$

$$= I_0 \frac{\sin^2\left(\pi \frac{b}{\lambda} \cdot \sin\Theta\right)}{\sin^2\left(\pi \frac{b}{N\lambda} \cdot \sin\Theta\right)}$$

with $x = \pi \frac{b}{\lambda} \cdot \sin\Theta$

$$= I_0 \cdot \frac{\sin^2(x)}{\sin^2(x/N)}$$

for $N \to \infty$, $\Delta b \to 0$

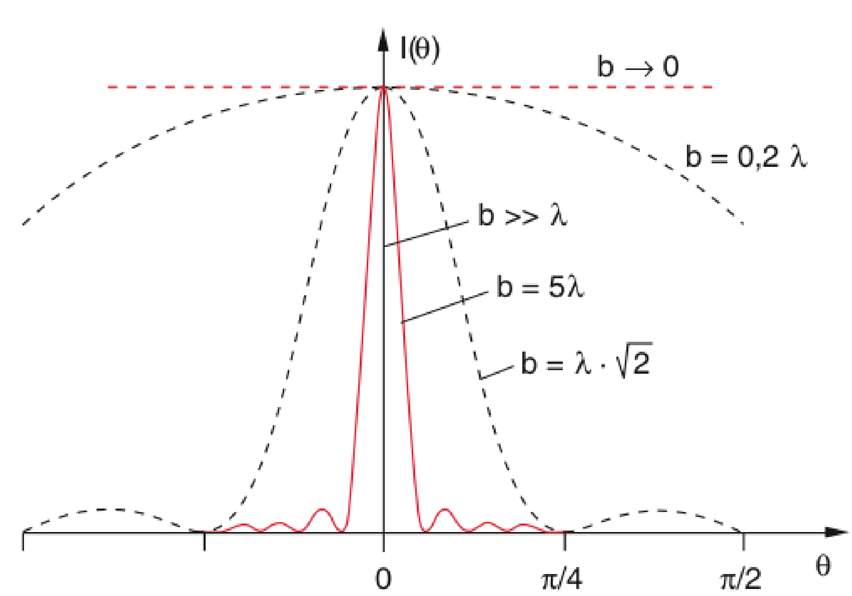
follows $\sin^2\left(\frac{x}{N}\right) \to \frac{x^2}{N^2}$

$\curvearrowright$ $I(\Theta) = N^2 I_0 \cdot \frac{\sin^2 x}{x^2} = I_s \frac{\sin^2(x)}{x^2}$

$\mathrm{sinc} = \frac{\sin x}{x}$ $\curvearrowright$ $I_s \cdot \mathrm{sinc}^2$

minima at $\pi \cdot \frac{b}{\lambda} \cdot \sin\Theta = m\pi$

$\curvearrowright$ $\underline{b \cdot \sin\Theta = m\lambda}$



$\sin\Theta_m = m \frac{\lambda}{b}$

for small Θ

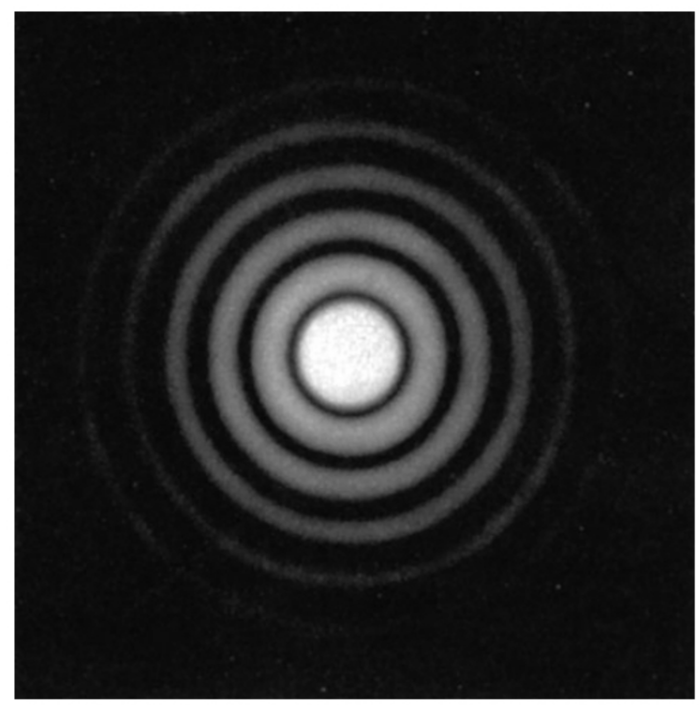
$\Theta_m \sim m \frac{\lambda}{b}$

the pattern gets wider with

- increasing λ
- decreasing b

Θ

<u>spherical aperture</u>



$$I(\Theta) = I_0 \cdot \left(\frac{2 J_1(x)}{x} \right)^2$$

$$x = \frac{2\pi R}{\lambda} \cdot \sin\Theta$$

J_1 Bessel function of first kind

with zeros at $x_1 = 1.22\pi$, $x_2 = 2.16 \cdot \pi$

↷ $I(\Theta)$ first zero at $\sin(\Theta_1) = \frac{0.61\lambda}{R}$

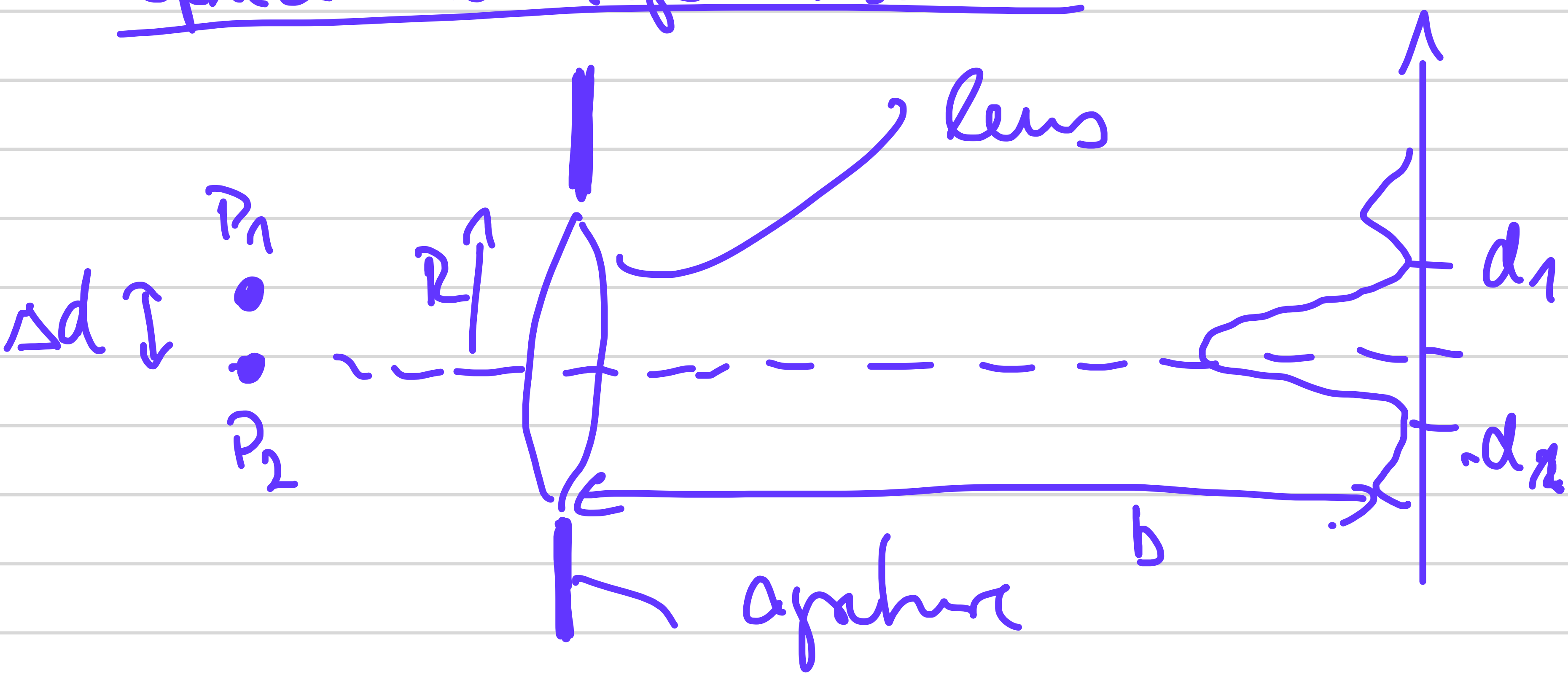
or $\sin(\Theta_1) = 1.22 \cdot \frac{\lambda}{D} \sim \Theta_1$

eye 150.000 cells / mm^2

↷ 2.58 μm distance

→ $b \cdot \sin\Theta_1 = 2.59\,\mu m$ for $D = 5$ mm, $b = 2$ cm

Optical Microscope resolution:



diffraction on aperture

single point P_1 creates diffraction pattern

$$I(\Theta) = I_0 \left(\frac{2 J_1(x)}{x}\right)^2$$

with $\quad x = \frac{2\pi}{\lambda} \cdot R \cdot \sin\Theta$

with $\quad x_1 = 1.22\,\pi$

$$\rightsquigarrow \quad 1.22\,\cancel{\pi} = \frac{2\cancel{\pi}}{\lambda} R \cdot \sin\Theta$$

$$\sin\Theta_1 = 0.61 \cdot \frac{\lambda}{R}$$

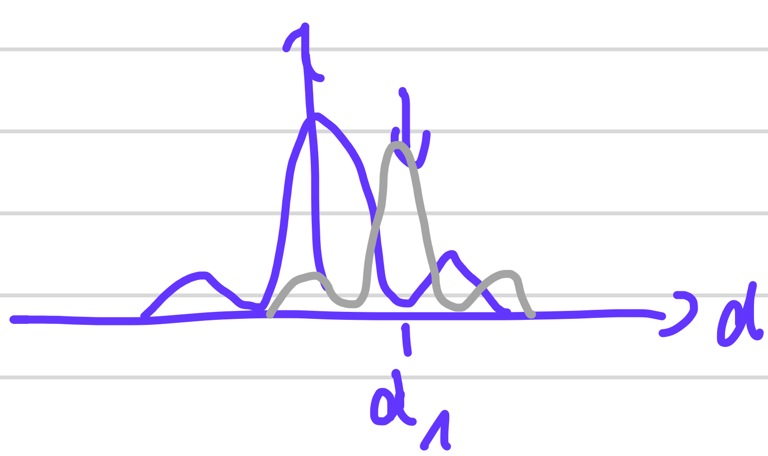
$$\frac{d}{b} \approx \sin\Theta_1 = 0.61 \cdot \frac{\lambda}{R}$$

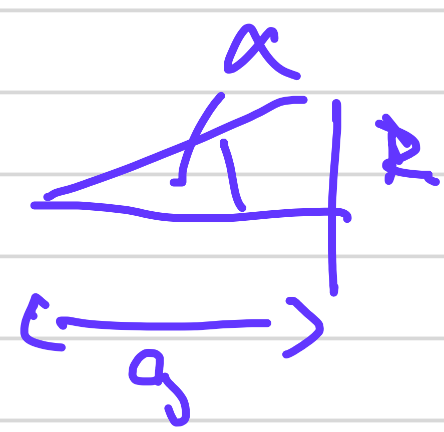
$$\rightsquigarrow \; d_1 = 0.61 \cdot \lambda \frac{b}{R} = 1.22 \cdot \lambda \frac{b}{D}$$

$$D = 2R$$

Rayleigh criterion:

both objects can be observed separately if the distance of the second object is d_1



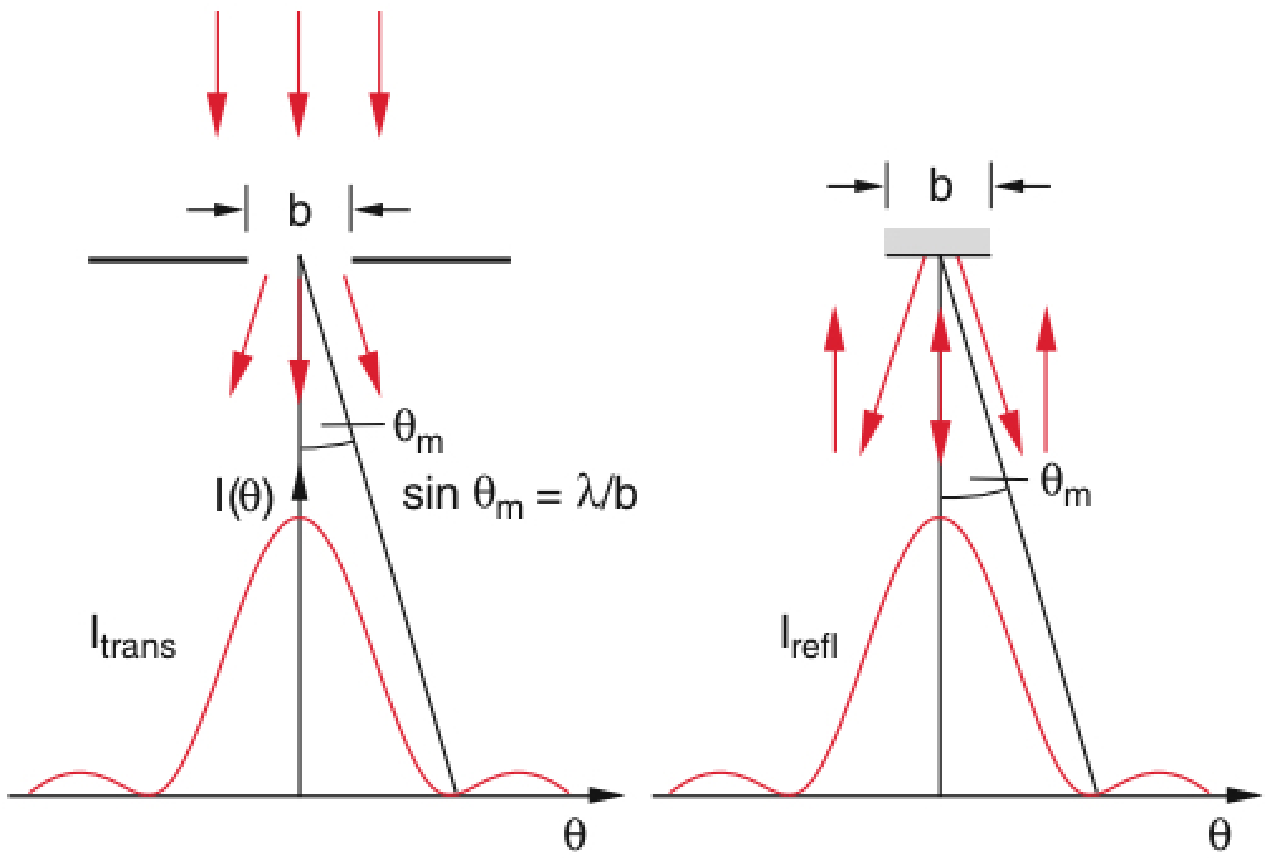


$$\rightsquigarrow \; \Delta d = d_1 \cdot \frac{g}{b} = 0.61 \, \lambda \frac{g}{R}$$

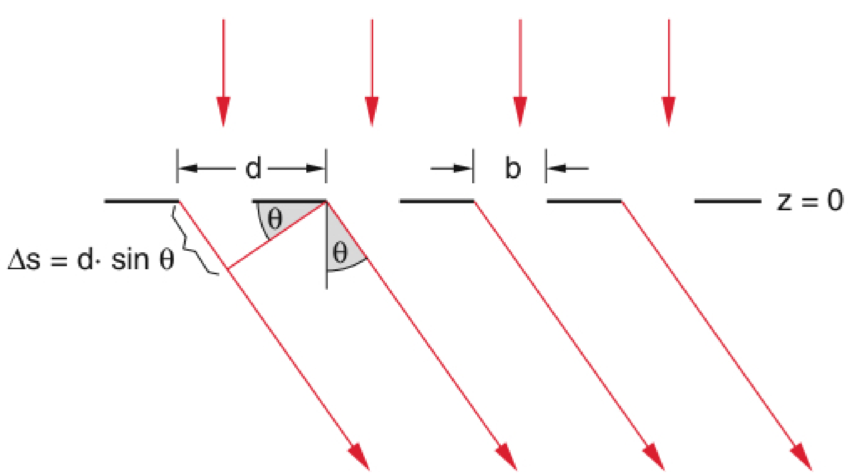
$$\sin(\alpha) \approx \frac{R}{g}$$

$$\rightsquigarrow \; \Delta d = 0.61 \cdot \frac{\lambda}{\sin\alpha}$$

$$= 0.61 \cdot \frac{\lambda}{NA} \, , \quad \underline{NA = n \cdot \sin(\alpha)}$$



Diffraction grating

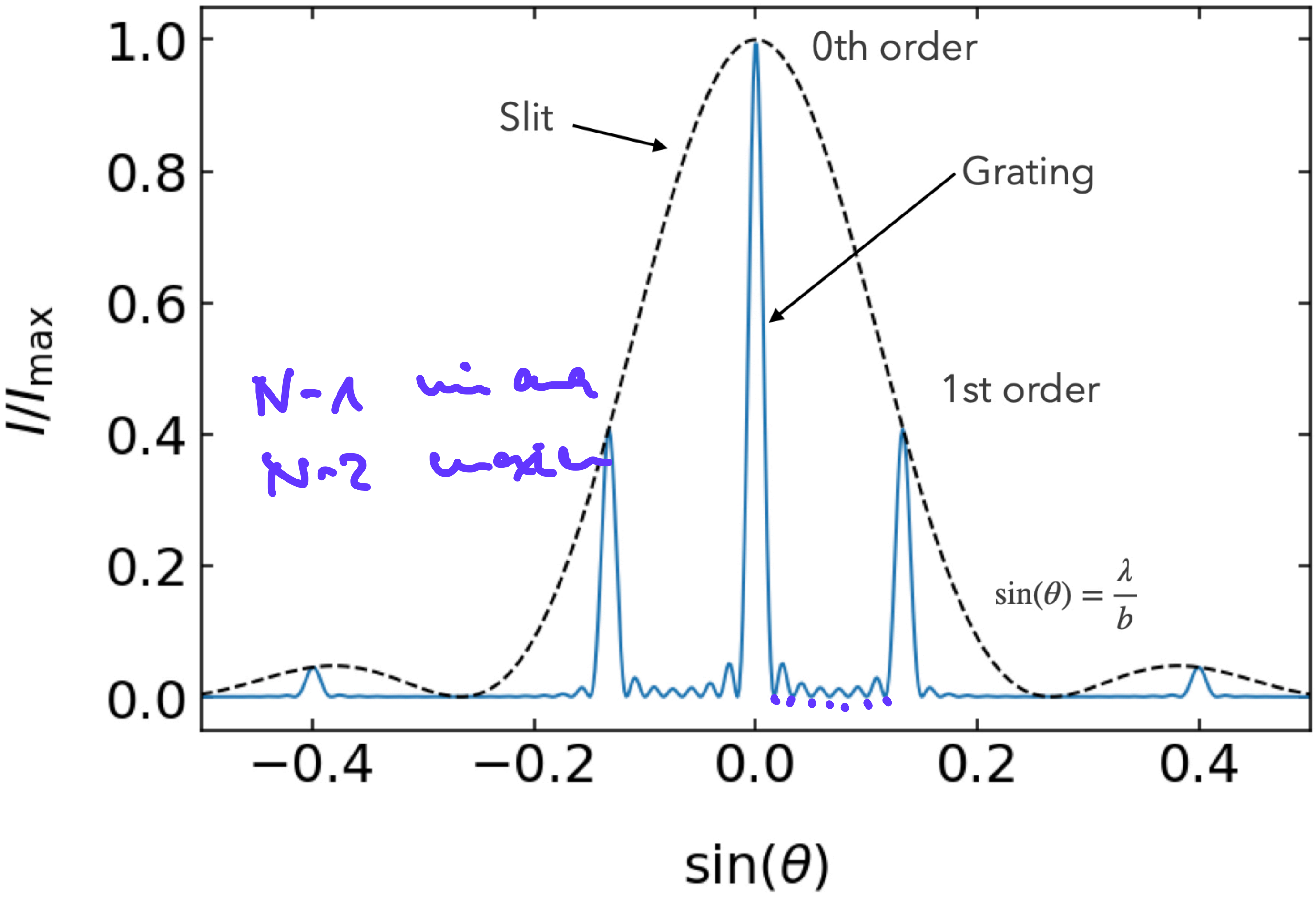


- diffraction by each slit
- interference from slits

$$I = I_S \cdot \frac{\sin^2\left(\pi \frac{b}{\lambda} \cdot \sin\Theta\right)}{\left[\pi \frac{b}{\lambda} \cdot \sin\Theta\right]^2} \; \frac{\sin^2\left[N \pi \frac{d}{\lambda} \sin\Theta\right]}{\sin^2\left[\pi \frac{d}{\lambda} \sin\Theta\right]}$$

I_S intensity transmitted by each slit

$N = 8 \quad \frac{d}{b} = 2$



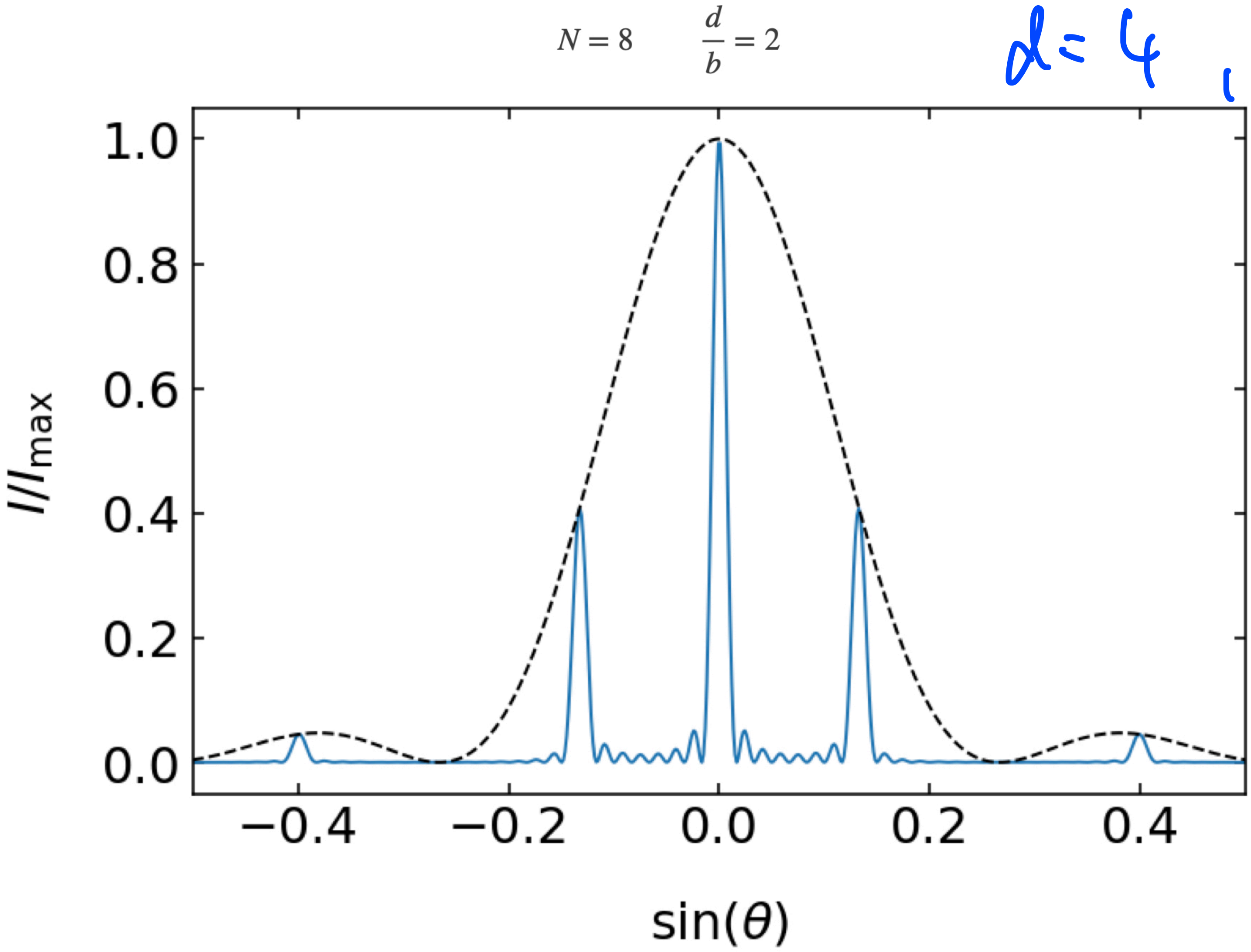
$$I = I_S \cdot \underbrace{\frac{\sin^2\left(\pi \frac{b}{\lambda} \cdot \sin\Theta\right)}{\left[\pi \frac{b}{\lambda} \cdot \sin\Theta\right]^2}}_{\text{slit}} \underbrace{\frac{\sin^2\left[N\pi \frac{d}{\lambda} \sin\Theta\right]}{\sin^2\left[\pi \frac{d}{\lambda} \sin\Theta\right]}}_{\text{grating}}$$

maxima:
grating

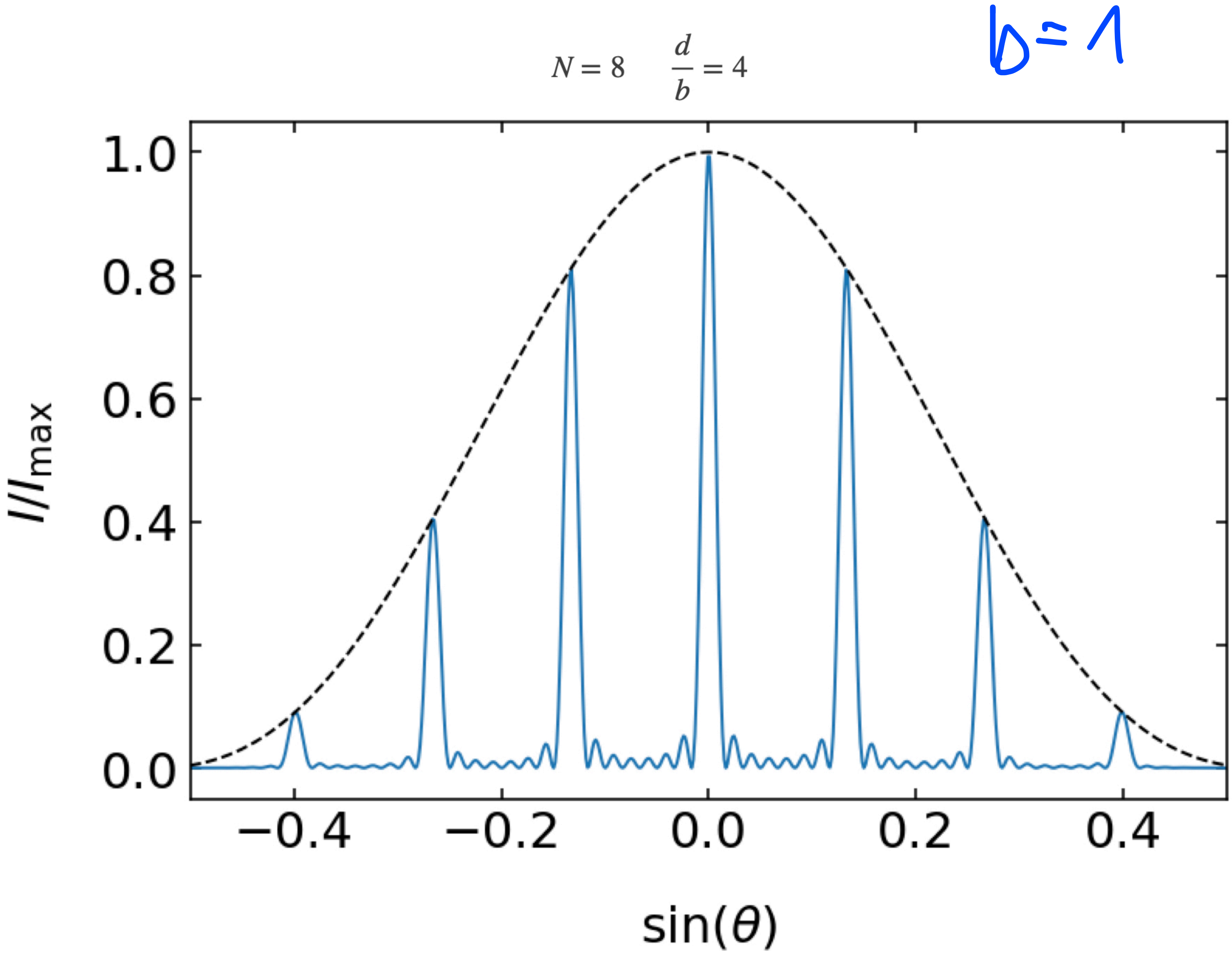
$d \cdot \sin\Theta = m\lambda$

m th order maxima

$d = 4$, $b = 2$

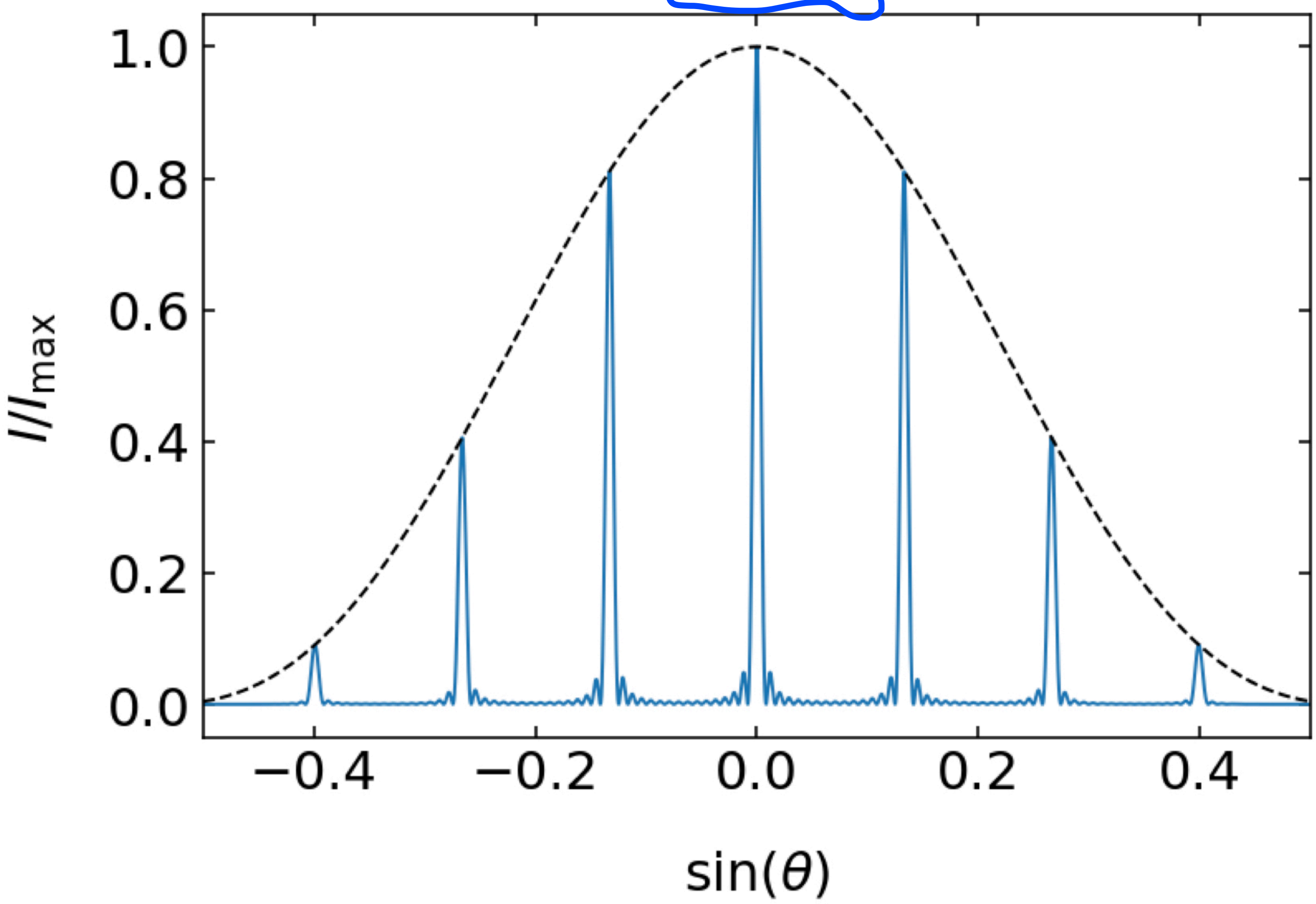


$b = 1$

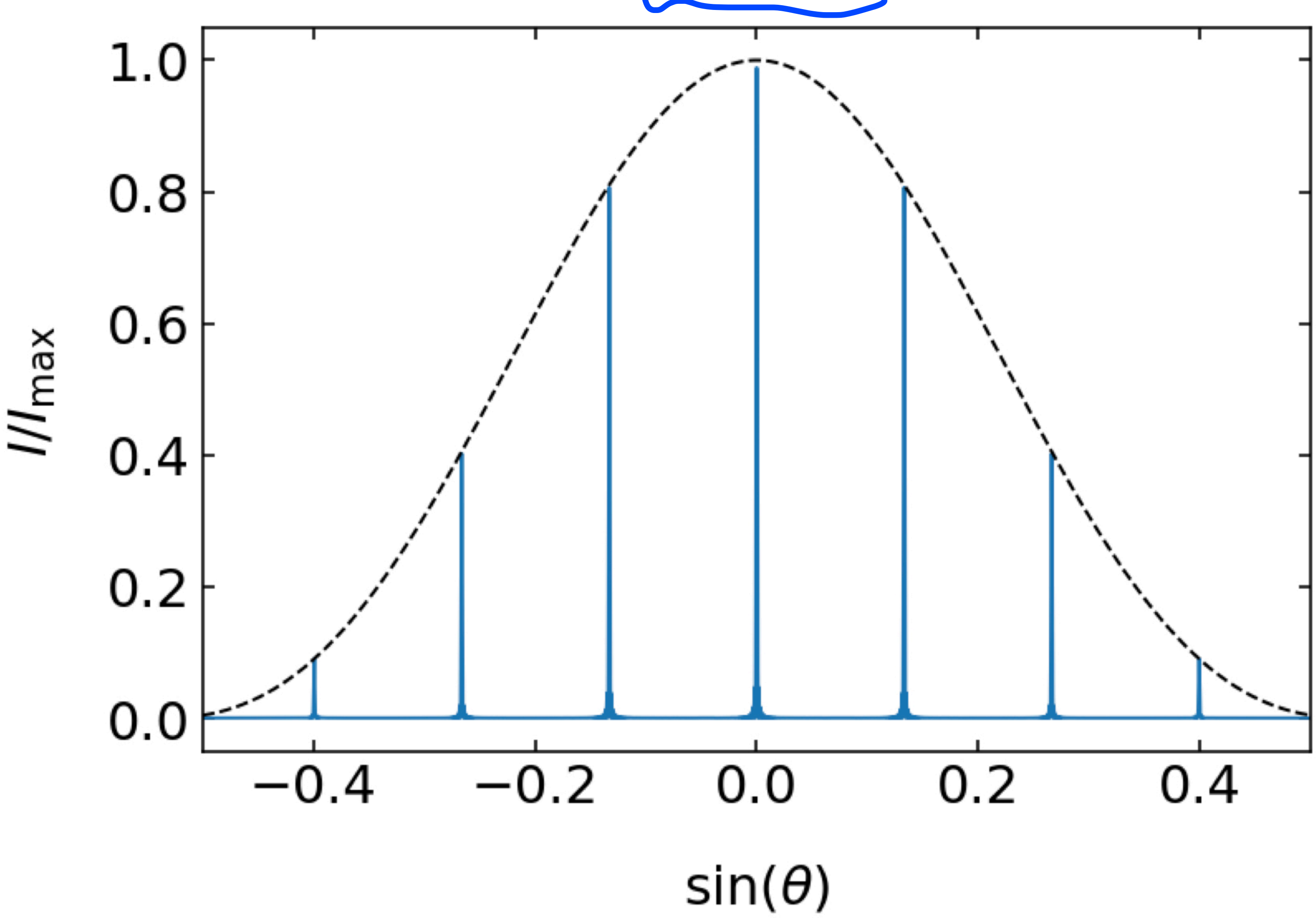


narrower slits make slit diffraction wider

$N = 16$


1.0
0.8
0.6
0.4
0.2
0.0
I/I_max
−0.4
−0.2
0.0
0.2
0.4
sin(θ)

$N = 100$


1.0
0.8
0.6
0.4
0.2
0.0
I/I_max
−0.4
−0.2
0.0
0.2
0.4
sin(θ)

$m = \frac{n}{N}$

$\rightsquigarrow$ next minimum is at $\frac{n+1}{N} \cdot \frac{\lambda_1}{d} = \sin(\Theta_1)$

this is the max for a diffrd WL

$\rightsquigarrow$ $\sin(\Theta_1) = m \frac{\lambda_2}{d}$

$\Rightarrow \left(m + \frac{1}{N}\right) \frac{\lambda_1}{\cancel{d}} = m \frac{\lambda_2}{\cancel{d}}$

$m \cdot \lambda_1 + \frac{\lambda_1}{N} = m \lambda_2$

$\frac{\lambda_1}{N} = m(\lambda_2 - \lambda_1) = m \Delta \lambda$

$\rightsquigarrow$ $\frac{\lambda}{\Delta\lambda} = m \cdot N = R$ resolving power of the grating

N is the number of illuminated slits!

grating analysis

minima (N-1):

$$\sin(\theta) \cdot N \pi \frac{d}{\lambda} = q \pi$$

$$\rightsquigarrow \sin(\theta) = \frac{q \cdot \lambda}{N \cdot d} \qquad q = \underset{\text{no}}{0}, 1, \dots \underset{\text{no}}{N}$$

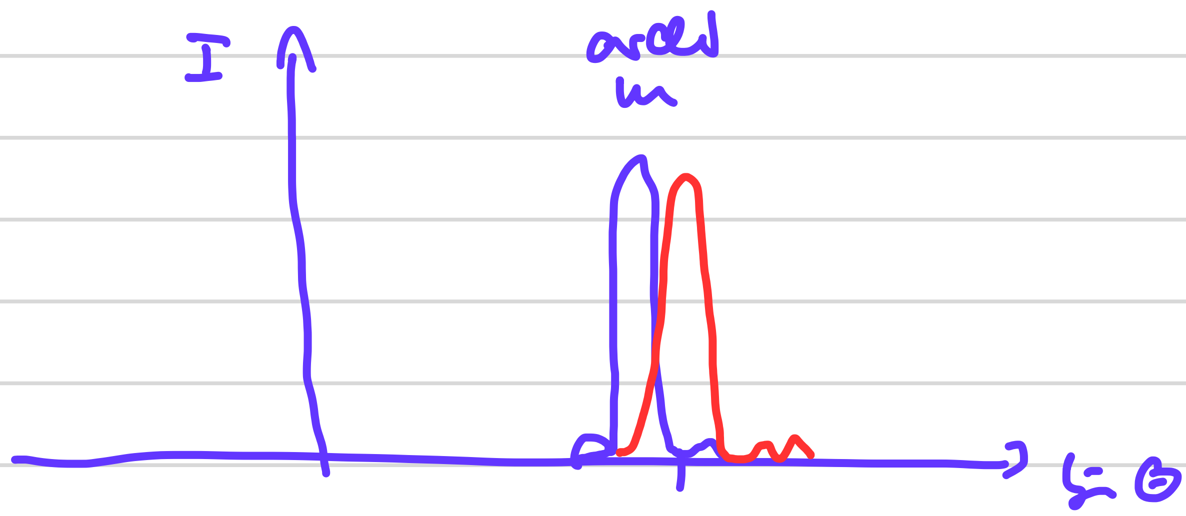
but only if $\sin(\theta) \neq m \cdot \frac{\lambda}{d}$

maxima (N-2):

$$\sin(\theta) N \pi \frac{d}{\lambda} = \frac{(2p+1)}{2} \pi$$

$$\rightsquigarrow \sin(\theta) = \frac{2p+1}{2N} \frac{\lambda}{d}$$

spectral resolution:

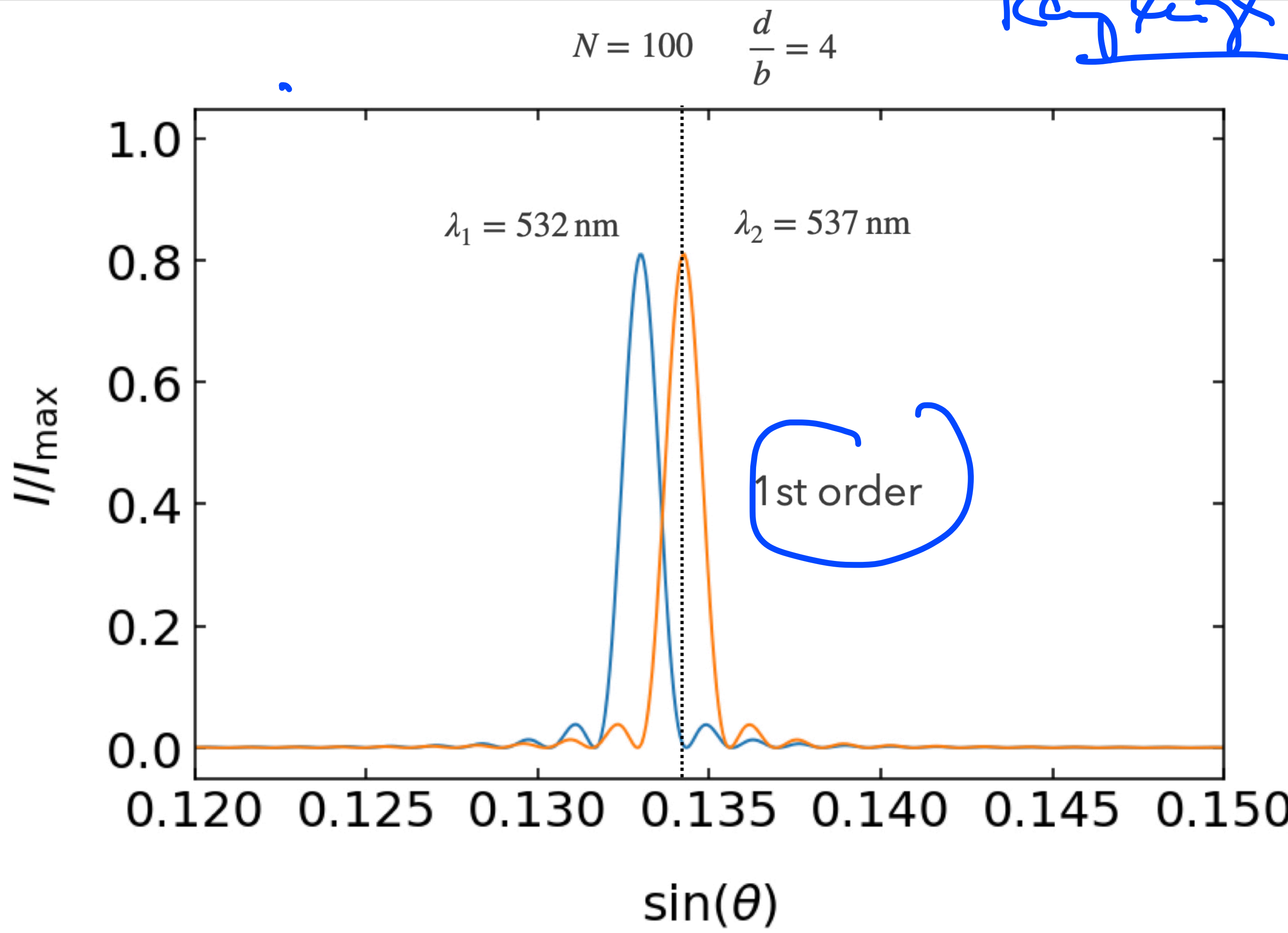


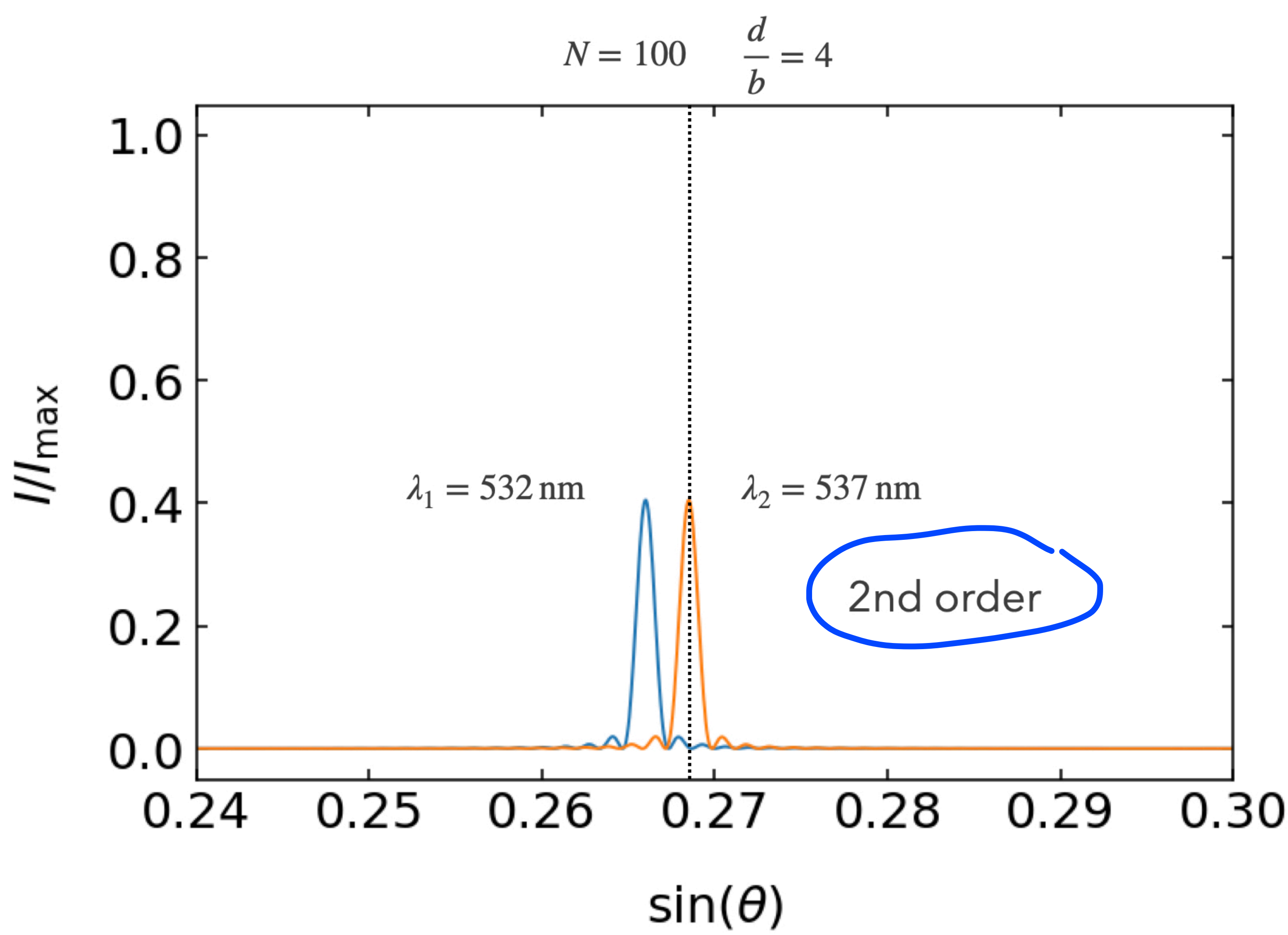
minimum of WL 1

lets say main max $\Rightarrow \sin(\theta) = m \frac{\lambda}{d}$

$$\sin(\theta) = \frac{q}{N} \frac{\lambda}{d}$$

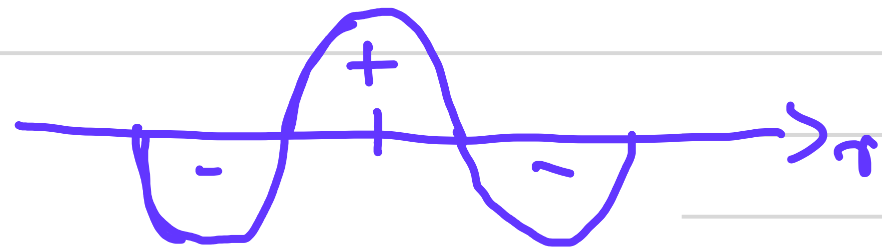
Rayleigh criter.

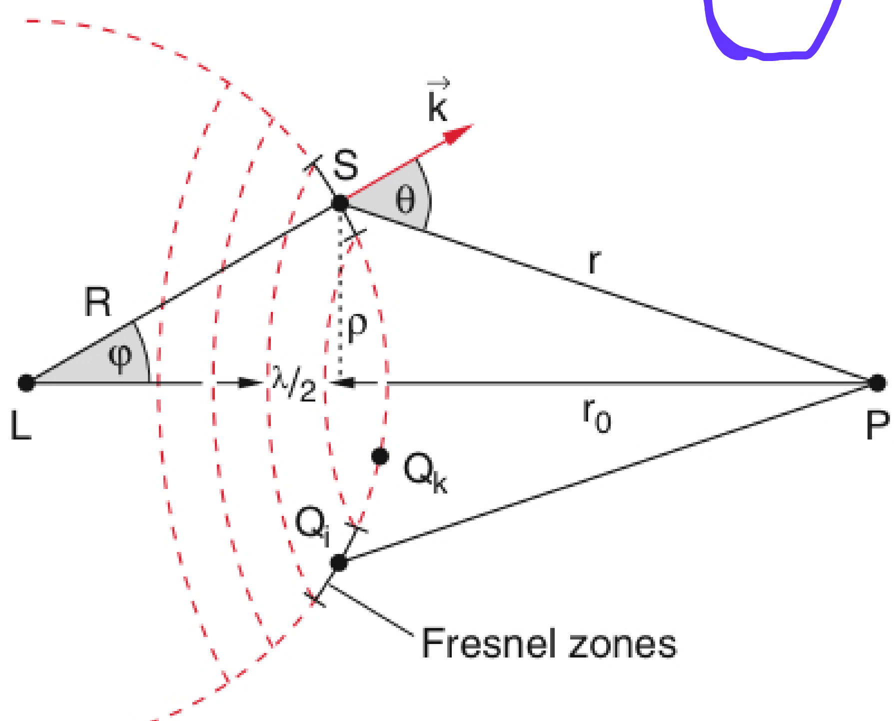




CD

Fresnel Zones





$r = SP$ with distance $\rho = R \cdot \sin\varphi$

$r(\varphi = 0) = r_0$

⇝ construct spheres with $r_m = r_0 + \frac{\lambda}{2} \cdot m$

from $r_m \to r_{m+1}$ Fresnel Zone

e.g. Q_i and Q_k differ by $\frac{\lambda}{2}$

⇝ destructive IF

$$\rho_m^2 = \left(r_0 + m\frac{\lambda}{2}\right)^2 - r_0^2$$

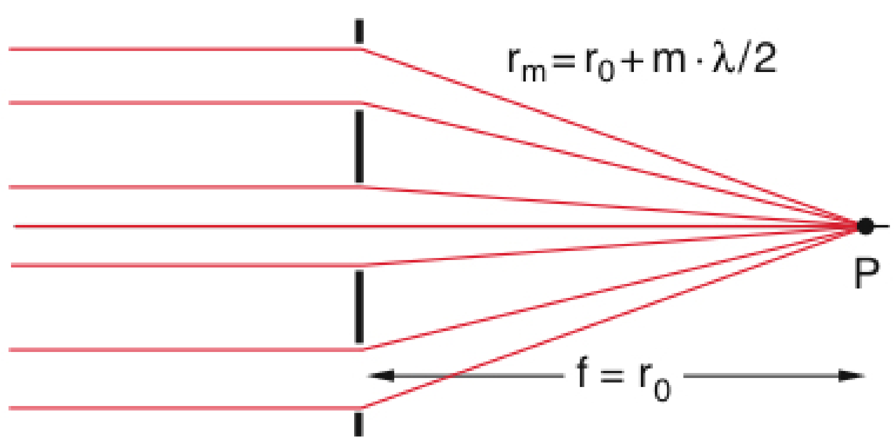
$$\rho_m^2 = r_0 m\lambda + m^2\lambda^2/4$$

for $r_0 \gg m\lambda$

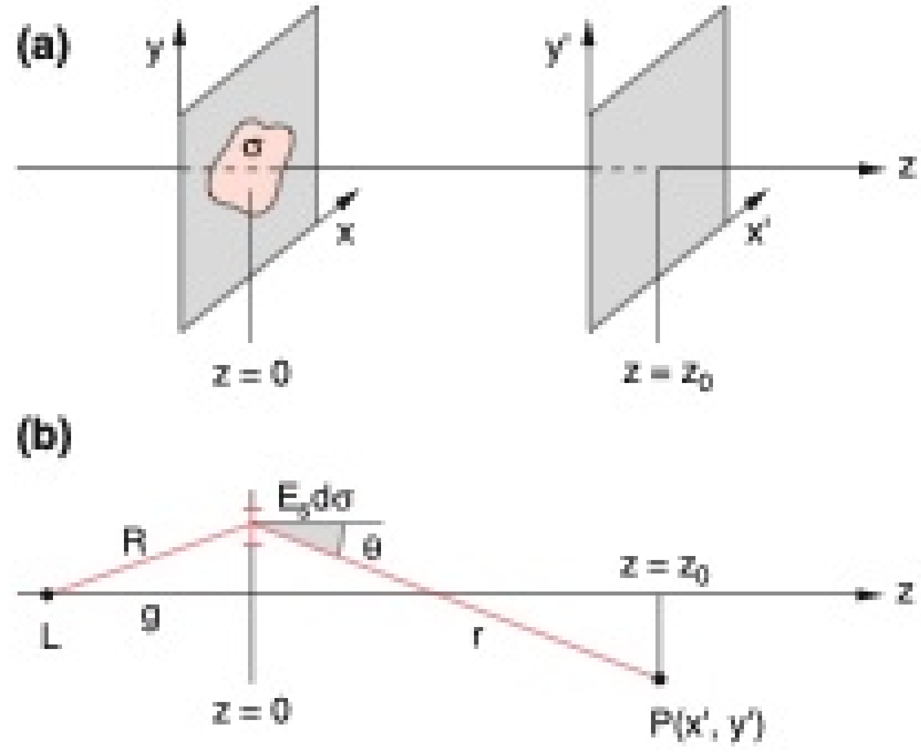
$$\leadsto \rho_m \approx \sqrt{m\, r_0 \cdot \lambda}$$

width of the m-th zone

$$\Delta\rho_m = \rho_{m+1} - \rho_m$$
$$= \sqrt{r_0\lambda}\,\left(\sqrt{m+1} - \sqrt{m}\right)$$



Diffraction integral



$$U_s(x,y) = U_0(x,y)\, e^{i\varphi(x,y)}$$

$$U_0(x,y) = \frac{A}{R} = \frac{A}{\sqrt{g^2+x^2+y^2}}$$

and $\varphi = (\omega t - kR)$

U_s is a Huygens wave at $z=0$

and contributes to $P(x',y')$

$$dU_p = C \cdot \frac{U_s\, d\sigma}{r} \cdot e^{-ikr}$$

$$C = i \cos\theta / \lambda$$

$$\boxed{\rightsquigarrow \; u_p = \iint C \cdot u_S \cdot \frac{e^{-ikr}}{r} \, dx \, dy}$$

$d\sigma = dx\, dy$

Fresnel Kirchhoff diff. integral

now

$$r = \sqrt{z_0^2 + (x-x')^2 + (y-y')^2} \nearrow \underline{\text{paraboloid}}$$

$$\approx z_0 \left(1 + \frac{(x-x')^2}{2z_0^2} + \frac{(y-y')^2}{2z_0^2} + \ldots \right)$$

neglect higher order terms

with $\cos(\Theta) = \frac{z_0}{r} \approx 1$, $C = \frac{i}{\lambda}$

$\rightsquigarrow$

$$u(x', y', z_0) = i \, \frac{e^{-ikz_0}}{\lambda z_0} \iint u_S(x, y)$$

$$\cdot \exp\left[-\frac{ik}{2z_0} (x-x')^2 + (y-y')^2 \right] dx \, dy$$

Fresnel Approximation

für Ka if aperture is small
the z_0

$$z_0 \gg \frac{1}{\lambda}(x^2+y^2) \quad \text{neglect } x^2, y^2$$

$$\leadsto r \approx z_0\left(1 - \frac{xx'}{z_0^2} - \frac{yy'}{z_0^2} + \frac{x'^2+y'^2}{2z_0^2}\right)$$

$$U(x', y', z_0) = A(x', y', z_0) \iint U_S(x, y)$$

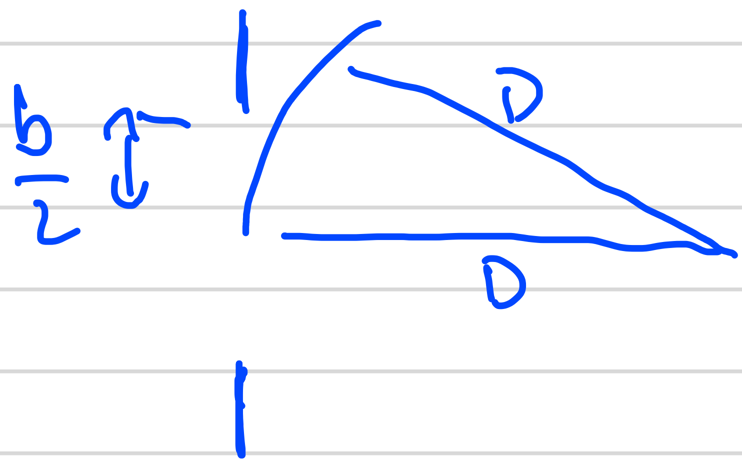
$$\cdot \exp\left[\frac{ik}{z_0}(x'x + y'y)\right] dx\, dy$$

with

$$A(x', y', z_0) = \frac{ie^{-ik_0 z}}{\lambda z_0} \cdot e^{-\frac{i\pi}{\lambda z}(x'^2+y'^2)}$$

Fraunhofer diffraction

Fraunhofer - Fresnel diffraction



$$D\sqrt{\frac{b^2}{D^2 4}+1} - D$$

$$= D\left(1+\frac{b^2}{8D^2} - \frac{b^4}{128\,D^4} + O(5)\right) - D \quad \text{for } D \gg b$$

$$\approx \frac{b^2}{8D}$$

path difference fine if $< \frac{\lambda}{8}$

$$\frac{b^2}{8D} < \frac{\lambda}{8}$$

$$\sim \frac{b^2}{\lambda D} < 1$$

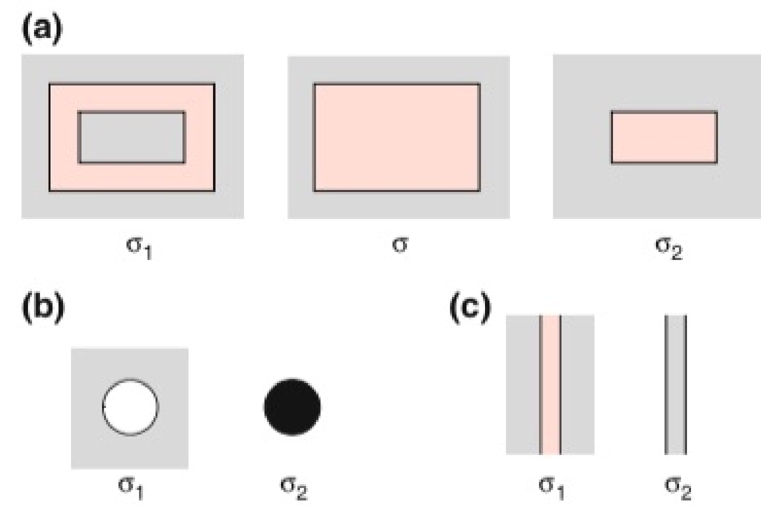
$$F = \frac{b^2}{\lambda D} \begin{cases} \ll 1 & \text{Fraunhofer} \\ \simeq 1 & \text{Fresnel} \\ \gg 1 & \text{geometric} \end{cases}$$

Fresnel number

Babinet Principle:

if screen can be subdivided in

$\sigma_1, \sigma_2, \ldots$



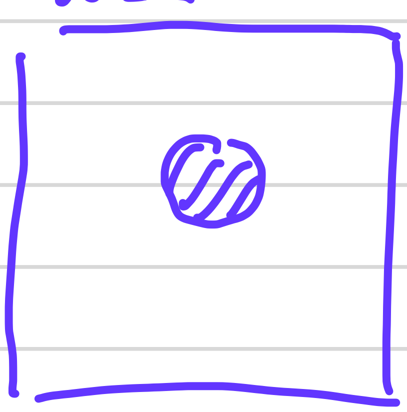
$$E_p(\sigma) = \sum_{i=1}^{N} E_p(\sigma_i)$$

hole

→ diffraction pattern

field u_h

disk



→ diffraction pattern

field u_d

$$u = u_h + u_d$$

but $u = 0$ for this case

⇝ $u_h = -u_d$

⇝ $I_h = I_d$ same diffraction pattern

3. Electromagnetic Waves

3.1. Electromagnetic Spectrum

$$\boxed{\text{show the spectrum again}}$$

so what is new?

$u(\vec{r},t) \to \vec{E}(\vec{r},t), \vec{B}(\vec{r},t)$

Maxwell - Equations

in vacuum without charges and currents

$\rho = 0, \vec{j} = 0$

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \qquad \vec{\nabla} \cdot \vec{E} = 0$$

$$\nabla \times \vec{B} = \varepsilon_0 \mu_0 \frac{\partial \vec{E}}{\partial t} \qquad \vec{\nabla} \cdot \vec{B} = 0$$

apply rot to eq. (1)

$$\nabla \times \nabla \times \vec{E} = -\nabla \times \frac{\partial \vec{B}}{\partial t}$$

$$= -\frac{\partial}{\partial t}(\nabla \times \vec{B})$$ ∇ not dep on time

$$= -\varepsilon_0 \cdot \mu_0 \frac{\partial^2 \vec{E}}{\partial t^2}$$

now $\nabla \times \nabla \times \vec{E} = \nabla(\nabla \cdot \vec{E}) - \nabla(\nabla \vec{E})$

$$= \underbrace{\text{grad}(\text{div}\,\vec{E})}_{=0} - \text{div}(\text{grad}\,\vec{E})$$

$\Rightarrow$ $\nabla \times \nabla \times \vec{E} = -\text{div}(\text{grad}\,\vec{E})$

$$\Delta \vec{E} = \varepsilon_0 \mu_0 \frac{\partial^2 \vec{E}}{\partial t}$$

or $$\boxed{\Delta \vec{E} - \varepsilon_0 \mu_0 \frac{\partial^2 \vec{E}}{\partial t^2} = 0}$$

this is a wave equation

where $c = \dfrac{1}{\sqrt{\varepsilon_0 \mu_0}} = \underline{\underline{3 \cdot 10^8 \frac{m}{s}}}$

↳ this means that static quantities ε_0, μ_0 determine the propagation of light !

$$\frac{\partial^2 E_x}{\partial x^2} + \frac{\partial^2 E_x}{\partial y^2} + \frac{\partial^2 E_x}{\partial z^2} = \frac{1}{c^2} \frac{\partial^2 E_x}{\partial t^2}$$

and the same for the other components of $\vec{E}$

same thing can now be done with eq. 2 for $\nabla \times \vec{B}$ giving a similar eq. for $\vec{B}$

3.2. Plane wave, spherical waves

a) monochromatic wave

$$\mathcal{E}(\vec{r},t) = \mathrm{Re}\{\underline{\vec{E}(\vec{r})}\,\underline{e^{-i\omega t}}\}$$

$$\mathcal{B}(\vec{r},t) = \mathrm{Re}\{\vec{B}(\vec{r})\,e^{-i\omega t}\}$$

$$\Delta\vec{E}(\vec{r}) + \frac{\omega^2}{c^2}E(\vec{r}) = 0 \qquad \text{H.H eq}$$

$\rightsquigarrow$ <u>into Maxwell</u>

$$\nabla\times\vec{E} = i\omega\vec{B}$$

$$\nabla\times\vec{B} = -\varepsilon_0\mu_0 i\omega\vec{E}$$

we look at $\vec{E}(\vec{r})$ for a plane wave

$$\vec{E}(\vec{r}) = \vec{E}_0\cdot e^{i\vec{k}\cdot\vec{r}} \qquad \nabla\times\vec{E}$$

$$\vec{B}(\vec{r}) = \vec{B}_0\cdot e^{i\vec{k}\cdot\vec{r}} \qquad \nabla\times\vec{B}$$

$$i\vec{k}\times\vec{E}_0 e^{i\vec{k}\cdot\vec{r}} = i\omega\vec{B}_0\cdot e^{i\vec{k}\vec{r}}$$

$$\vec{k}\times\vec{E}_0 = \omega\vec{B}_0 \qquad \vec{B}_0\perp\vec{E}_0$$

$$\vec{B}_0\perp\vec{k}$$

$$\nabla \times \vec{B} = -i\omega \varepsilon_0 \mu_0 \vec{E}$$

$$i\vec{k} \times \vec{B}_0 \cancel{e^{i\vec{k}\vec{r}}} = -i\omega \varepsilon_0 \mu_0 \vec{E}_0 \cancel{e^{i\vec{k}\vec{r}}}$$

$$\vec{k} \times \vec{B}_0 = -\omega \varepsilon_0 \mu_0 \vec{E}_0$$

$$\vec{k} \times \vec{B}_0 = -\frac{\omega}{c^2} \vec{E}_0$$

$\vec{E}_0 \perp \vec{B}_0$

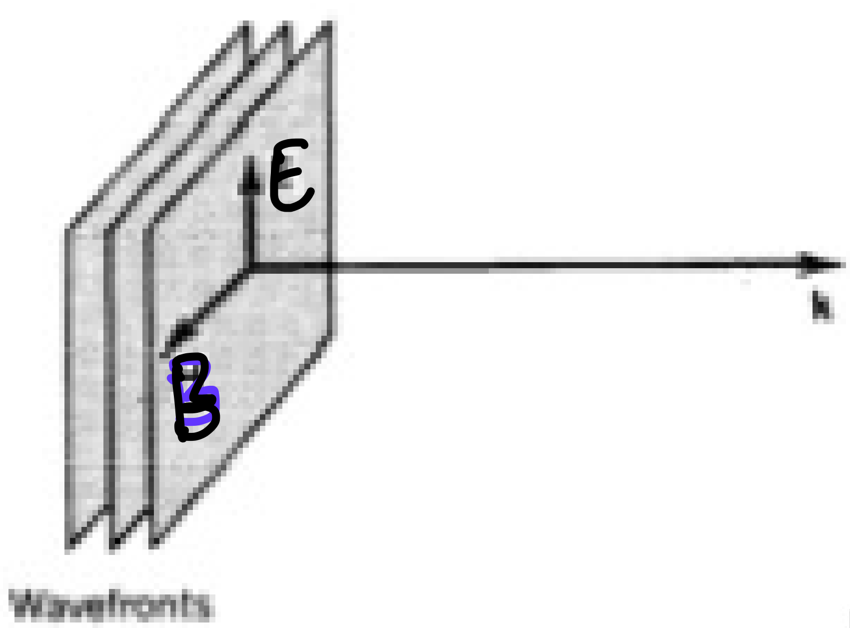
$\vec{E}_0 \perp \vec{k}$

$$|k| \cdot |\vec{B}_0| = \frac{\omega}{c^2} |\vec{E}_0|$$

$k = \frac{\omega}{c}, \; c = \frac{\omega}{k}$

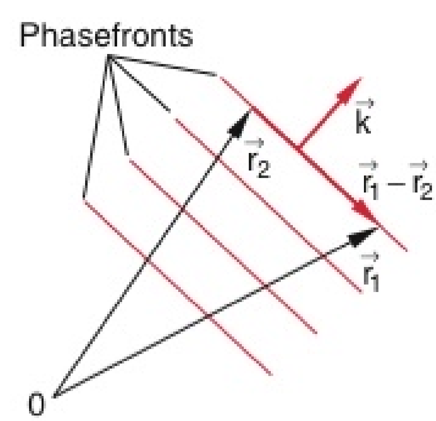
$$\Rightarrow \boxed{B_0 = \frac{1}{c} E_0}$$

both amplitudes are connected by factor c



transverse wave

plane wave



$\vec{k} \cdot (\vec{r}_1 - \vec{r}_2) = 0$
$\Rightarrow \vec{k} \cdot \vec{r} = \text{const}$
for all points of
a plane perpendicular
to k

Spherical Wave

it is a bit more complicated here

$\vec{A}(\vec{r}) = A_0 \cdot U(\vec{r})\,\hat{x}$ aux. field

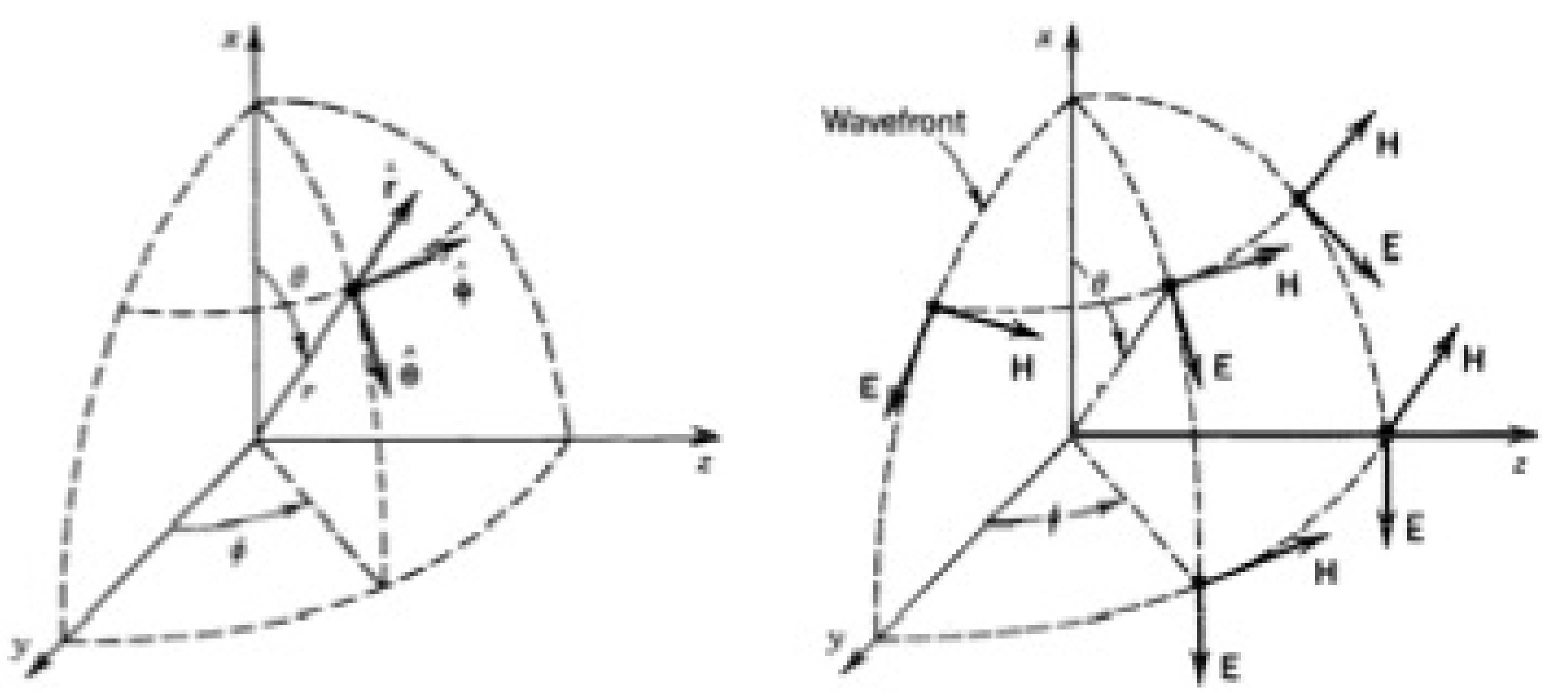
$U(\vec{r}) = \frac{1}{r} e^{-ikr}$

$\vec{A}(\vec{r})$ satisfies $\nabla^2 \vec{A} + k^2 \vec{A} =$

for $r \gg \lambda$ or $kr \gg 2\pi$

$\vec{E}(\vec{r}) = E_0 \cdot \sin\theta \cdot U(\vec{r})\,\hat{\theta}$

$\vec{B}(\vec{r}) = B_0 \cdot \sin\theta\, U(\vec{r})\,\hat{\phi}$



3.3. Polarization of EM Waves

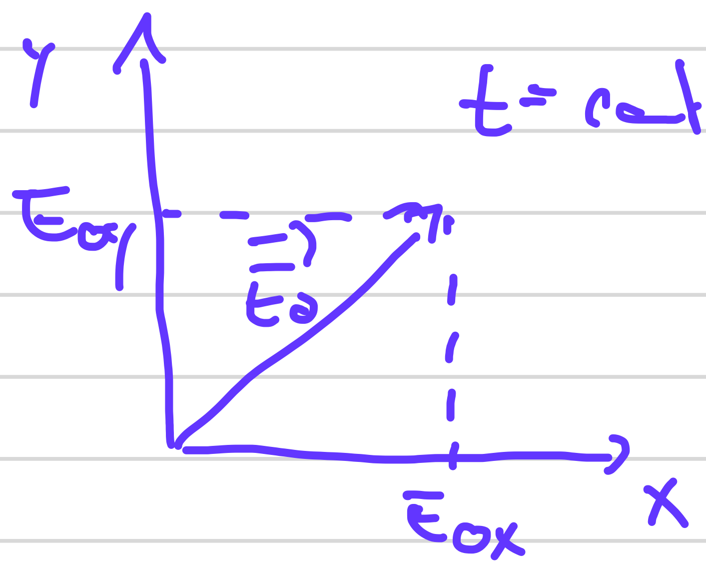
The polarization of electromagnetic waves is defined by the direction of the electric field vector

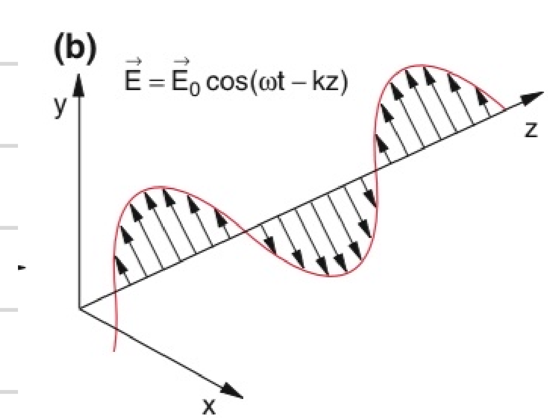
3.3.1. Linearly Polarized Waves

-> $\vec{E}_0$ points always in the same direction

e.g. $\vec{E} = \vec{E}_0 \cdot e^{i(\omega t - kz)}$ along z

$$\vec{E}_0 = E_{0x} \cdot \hat{e}_x + E_{0y} \cdot \hat{e}_y$$





↳ linearly pol. wave has two components

$$E_x = E_{0x}\, e^{i(\omega t - kz)}$$

$$E_y = E_{0y} \cdot e^{i(\omega t - kz)}$$

→ both oscillate in phase

3.3.2 Circular Polarized Light

both vectorical components

E_{0x} and E_{0y} are equal

↳ $E_{0x} = E_{0y} = E_0$

e.g. $$E_x = E_{0x} \cdot e^{i(\omega t - kz)}$$

$$E_y = E_{0y} \cdot e^{i(\omega t - kz \pm \frac{\pi}{2})}$$

↳ makes it sine funct.

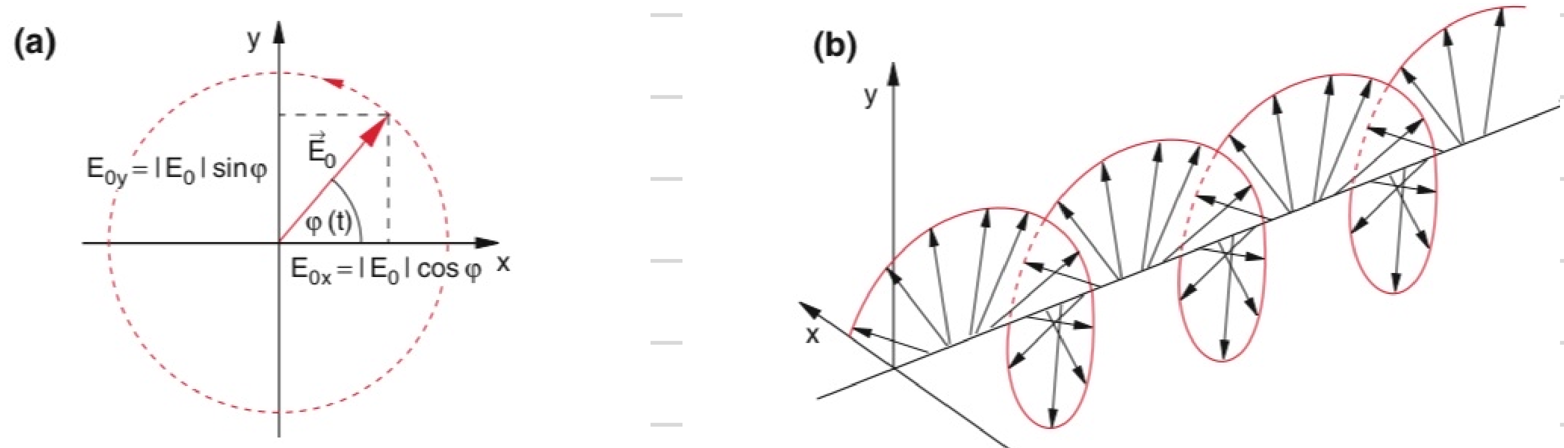
$$E_x = E_{0x} \cdot \cos(\omega t - kz)$$

$$E_y = E_{0y}\, \sin(\omega t - kz)$$

$$\vec{E} = \vec{E}_x \cdot \hat{e}_x + \vec{E}_y \, \hat{e}_y$$

check the vector as a function of time at $z=0$ in the x-y plane

$$\vec{E} = E_0 \cdot \cos(\omega t)\, \hat{e}_x + E_0 \cdot \sin(\omega t)\, \hat{e}_y$$

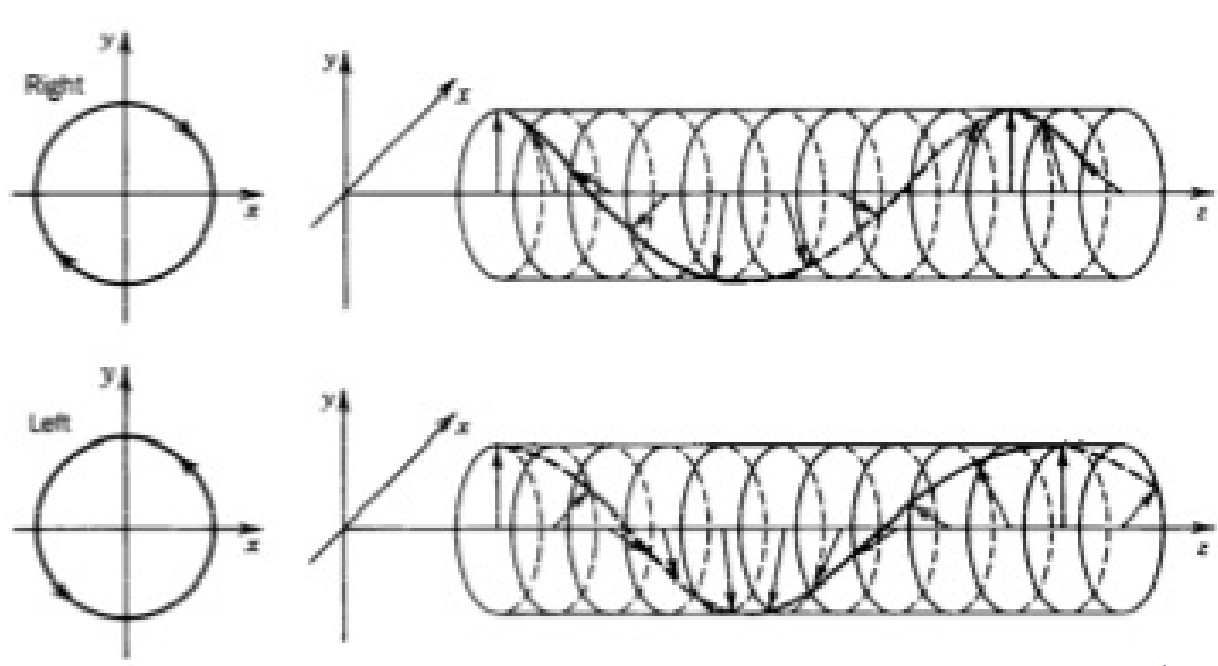


left circular

rotation direction can be either clockwise or counter-clockwise in the direction of propagation

σ^+

σ^-



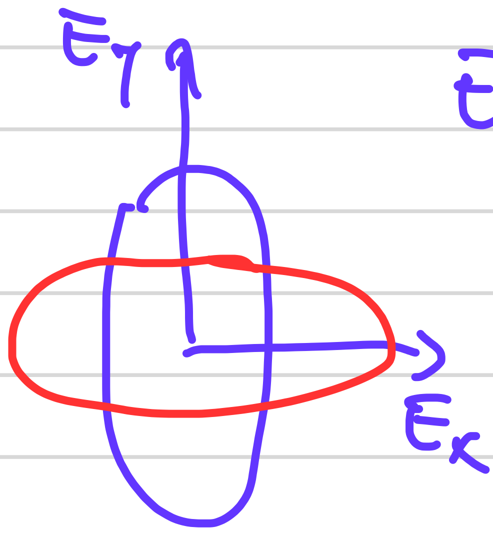
corresponds to spin angular momentum in QM

3.3.3 Elliptically Polarised light

for $E_{0x} \neq E_{0y}$ and $\varphi = \pm \frac{\pi}{2}$

$\Rightarrow$ elliptic but also for

$E_{0x} \neq E_{0y}$ and $\Delta\varphi \neq \frac{\pi}{2}$



3.3.4 Unpolarised light

- statistical changes in the direction of the E-vector
- depends on the way light is emitted